Business problem

Steps to maximize profit:

Step 1: Determine demand equation.

$\cancel{q = A \cdot p + B}$ $\quad p = Aq + B$

Data point: pair (p, q)

Our data pts: $(p_1, q_1) = (2, 3000)$

$(p_2, q_2) = (1.90, 3100)$

$$\text{slope} = A = \frac{p_1 - p_2}{q_1 - q_2} = \frac{2 - 1.90}{3000 - 3100} = \frac{0.1}{-100} = -\frac{1}{1000}$$

$$\Rightarrow \quad p = -\frac{1}{1000} q + B$$

$$\Rightarrow 2 = -\frac{1}{1000} 3000 + B \Rightarrow B = 5$$

demand equation: $\boxed{p = -\frac{1}{1000} q + 5}$

Step 2: Write the revenue function

$$\boxed{R(q) = p \cdot q = \left(-\frac{1}{1000}q + 5\right) \cdot q = -\frac{1}{1000}q^2 + 5q}$$

Step 3: Write cost function:

$$C(q) = F + V(q)$$

$$\boxed{C(q) = 3250 + 0.75 \cdot q}$$

Step 4: Write the profit function:

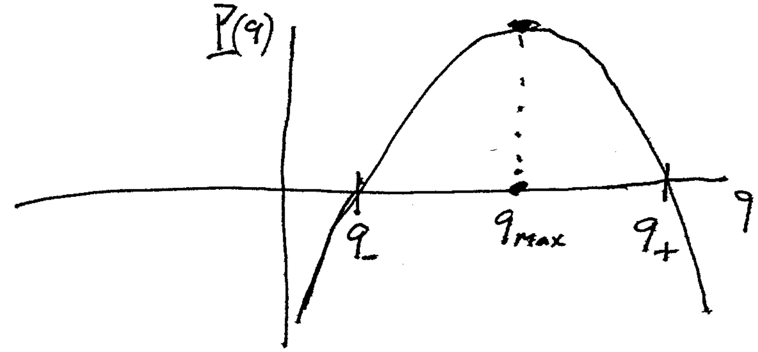
$$P(q) = R(q) - C(q)$$

$$= -\frac{1}{1000}q^2 + 5q - (3250 + 0.75q)$$

$$= \frac{-q^2 + 5000q - 3{,}250{,}000 - 750q}{1000}$$

$$\boxed{P(q) = \frac{-q^2 + 4250q - 3{,}250{,}000}{1000}}$$

Step 5: Maximize profit



$$q_{max} = \frac{q_- + q_+}{2}$$

q_-, q_+ satisfy $P(q_\pm) = 0$

$$\frac{-q^2 + 4250q - 3,250,000}{\cancel{1000}} = 0$$

$$\Rightarrow q^2 - 4250q + 3250000 = 0$$

quadratic formula: $$q_\pm = \frac{4250 \pm \sqrt{(4250)^2 - 4 \cdot 3250000}}{2}$$

$$= \underset{q_-}{\underset{\parallel}{1000}} \;\;\&\;\; \underset{q_+}{\underset{\parallel}{3250}}$$

$$\boxed{q_{max} = \frac{q_- + q_+}{2} = 2125}$$

$$\boxed{\underline{P}(q_{max}) = \underline{P}(2{,}125) = 1{,}265.625}$$

More steps:

$$\underline{P}(q_{max}) = \underline{P}(2125) = \frac{-2125^2 + 4250 \cdot 2125 - 3250000}{1000}$$

$$= \boxed{1265.625}$$

Price that maximizes profit (from demand equation):

$$p_{max} = -\frac{1}{1000} q_{max} + 5 = -\frac{1}{1000} \cdot 2125 + 5 = -2.125 + 5$$

$$= \boxed{2.875 \,\$}$$