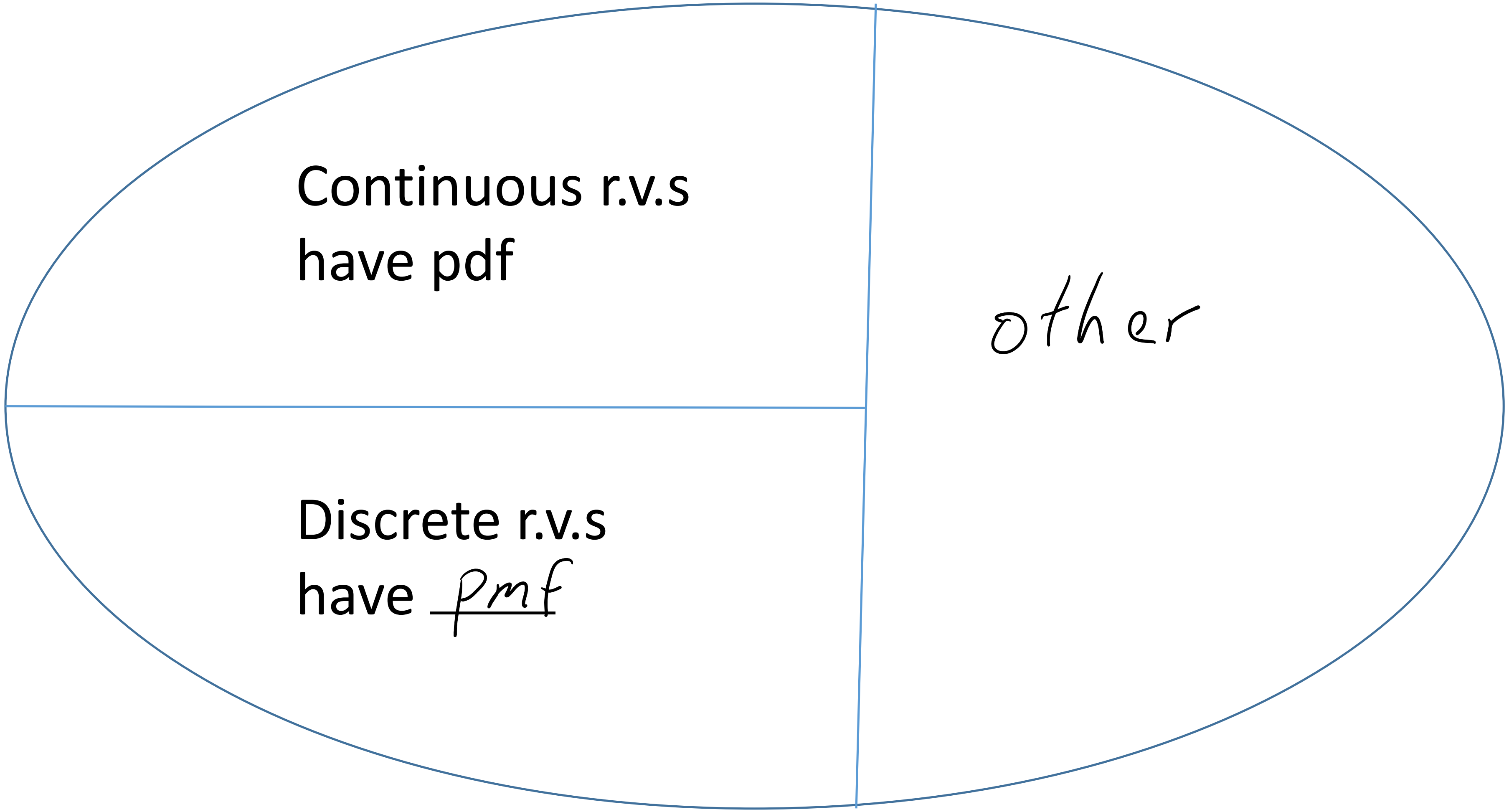
Section 3.2

Cumulative distribution function

Kinds of random variables (and how their probability dist. can be characterized)



Cumulative distribution function (cdf)

Goal: A function which characterizes the probability dist. for any r.v., continuous, discrete, or other.

Def: The cumulative distribution function (cdf), F, of a random variable X satisfies

$$\underline{F(s)} = P(X \leq s) \quad \text{for all } s \in \mathbb{R}.$$

Fact: The cdf completely characterizes the probability dist. of a r.v.

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Meaning: Given just the cdf, we can always determine $P(X \in B)$.

Ex) Given cdf: $F(s)$, $s \in \mathbb{R}$, what is $P(X \in (s,t])$?

$$\underbrace{P(X \le t)}_{F(t)} = P(\{X \le s\} \cup \{X \in (s,t]\})$$

$$= \underbrace{P\{X \le s\}}_{F(s)} + P(X \in (s,t])$$

disjoint events

Thus, $P(X \in (s,t]) =$ $F(t) - F(s)$

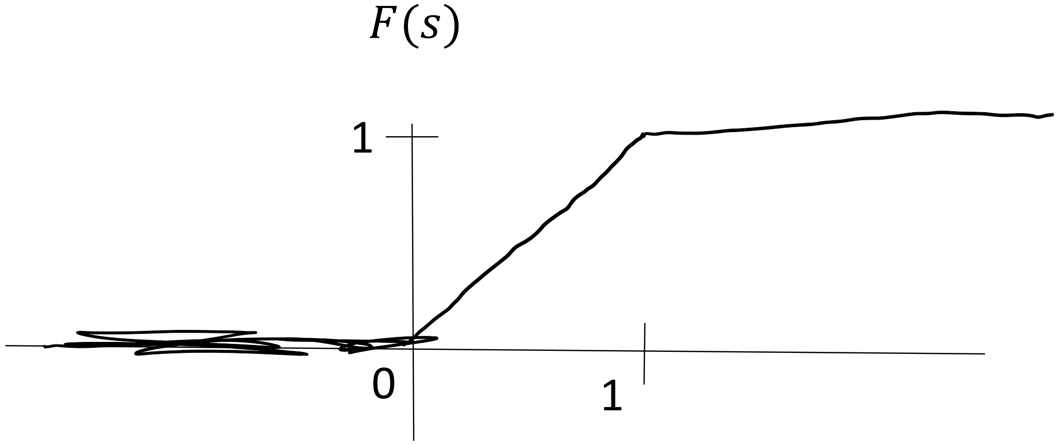
Ex) $X \sim Unif[0,1]$

$P(X \leq s) = \underline{\quad 0 \quad}$ for $s < 0$,

$P(X \leq s) = \underline{\quad s \quad}$ for $s \in [0,1]$,

$P(X \leq s) = \underline{\quad 1 \quad}$ for $s > 1$,

$$F(s) = \begin{cases} 0 & s < 0 \\ s & s \in [0,1] \\ 1 & s > 1 \end{cases}$$



Cdf for continuous r.v.

Fact: If X is continuous, with pdf f, then the cdf satisfies:

$$F(s) = P(X \leq s) = \int_{-\infty}^{s} f(x)\,dx$$

Ex) Suppose X has pdf $f(x) = \begin{cases} e^{-x} & \text{for } x \geq 0 \\ 0 & \text{for } x < 0 \end{cases}$

Then, for $s < 0$, $F(s) = \int_{-\infty}^{s} 0\,dx = 0$

For $s > 0$, $F(s) = \int_{-\infty}^{s} f(x)\,dx = \int_{0}^{s} f(x)\,dx = \int_{0}^{s} e^{-x}\,dx = -e^{-x}\Big|_{0}^{s}$

$= \boxed{1 - e^{-s}}$

Finding pdf from cdf (for continuous r.v.)

Recall: $F(s) = \int_{-\infty}^{s} f(x)\,dx$

Thus, $f(s) =$ $\underline{F'(s)}$

Ex) $F(s) = \begin{cases} 1, & s > 1 \\ s^2, & s \in [0,1] \\ 0, & s < 0 \end{cases}$

$\underline{s \leq 0}$

$f(s) = \frac{d}{ds} 0 = 0$

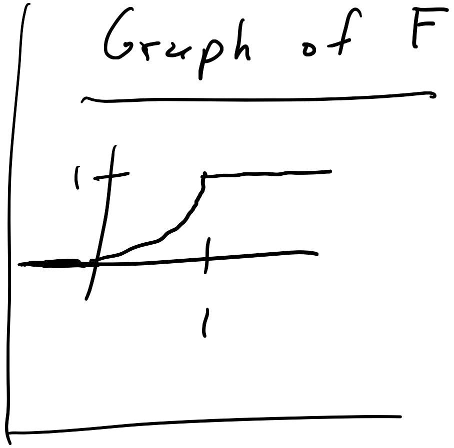
$\underline{s \in [0,1]}$

$f(s) = \frac{d}{ds} s^2 = 2s$

$\underline{s > 1}$

$f(s) = \frac{d}{ds} 1 = 0$

$$\boxed{f(s) = \begin{cases} 0 & s < 0 \\ 2s & s \in [0,1] \\ 0 & s > 1 \end{cases}}$$



Cdf for discrete r.v.

Ex) $X =$ # of heads on two coin flips

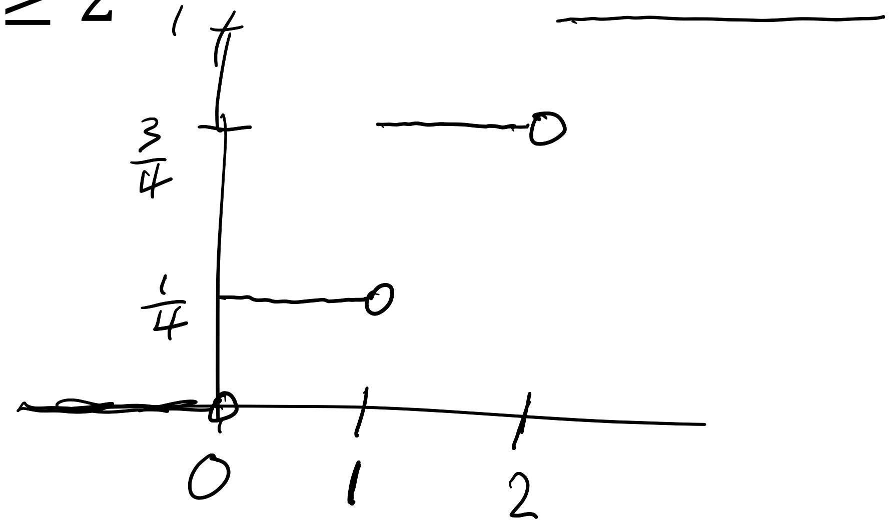
$P(X \leq s) =$ $\underline{\quad 0 \quad}$ for $s < 0$

$P(X \leq s) =$ $\underline{p(0) = \frac{1}{4}}$ for $s \in [0,1)$

$P(X \leq s) =$ $\underline{p(0) + p(1) = \frac{3}{4}}$ for $s \in [1,2)$

$P(X \leq s) =$ $\underline{\quad 1 \quad}$ for $s \geq 2$

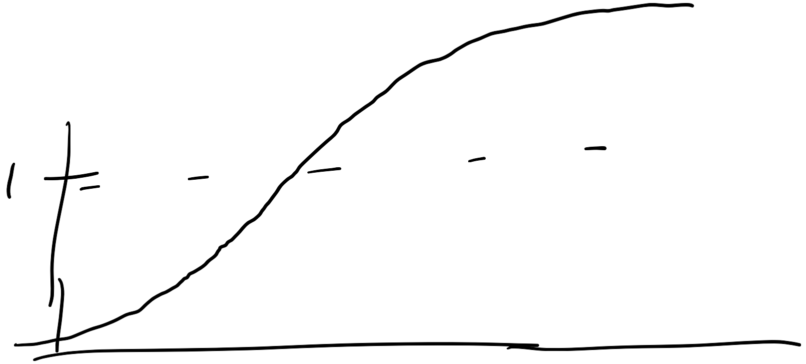
$$F(s) = \begin{cases} 0 & s < 0 \\ \frac{1}{4} & s \in [0,1) \\ \frac{3}{4} & s \in [1,2) \\ 1 & s \geq 2 \end{cases} =$$



Identify graph of cdf

cont. disc. not cdf

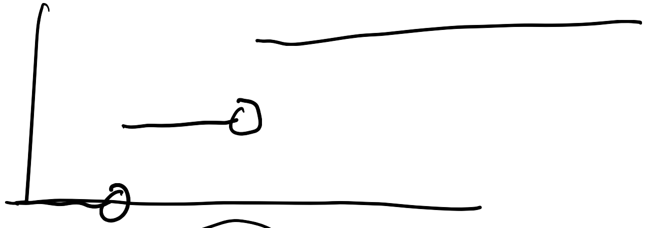
cdf cannot decrease



cont. disc. not cdf

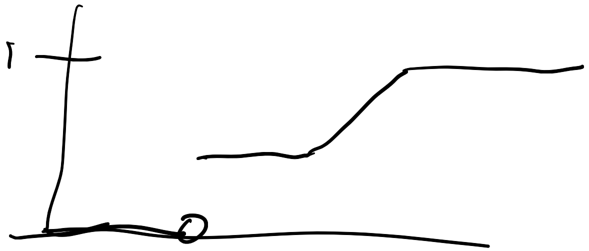
cdf cannot be > 1

cont. disc. not cdf



cont disc not cdf

Finding pmf given cdf

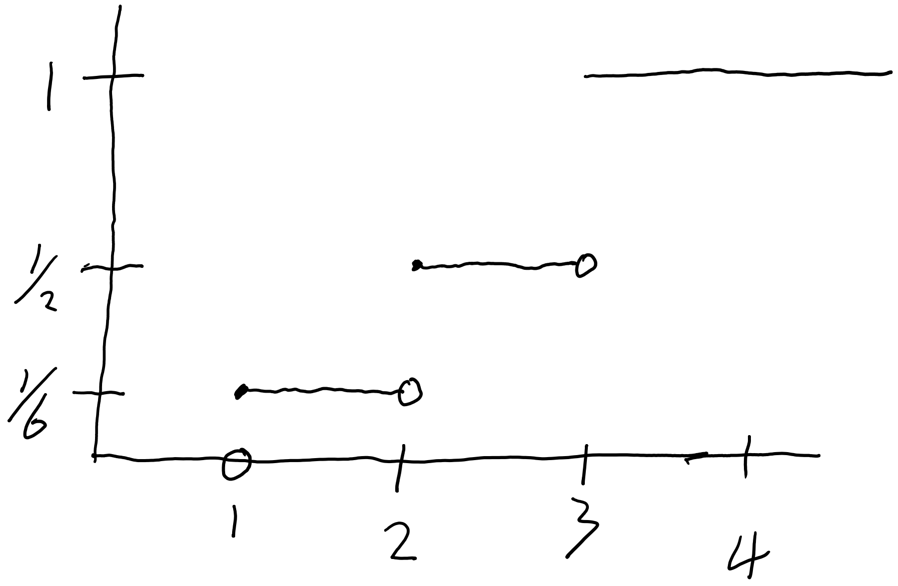


not disc.
or cont.

Fact: Suppose r.v. X has cdf F which is piecewise constant. Then X is a discrete r.v. The values that X can take are the places where F has jumps. If x is such a point, then $P(X = x)$ is the size of the jump.

Ex)

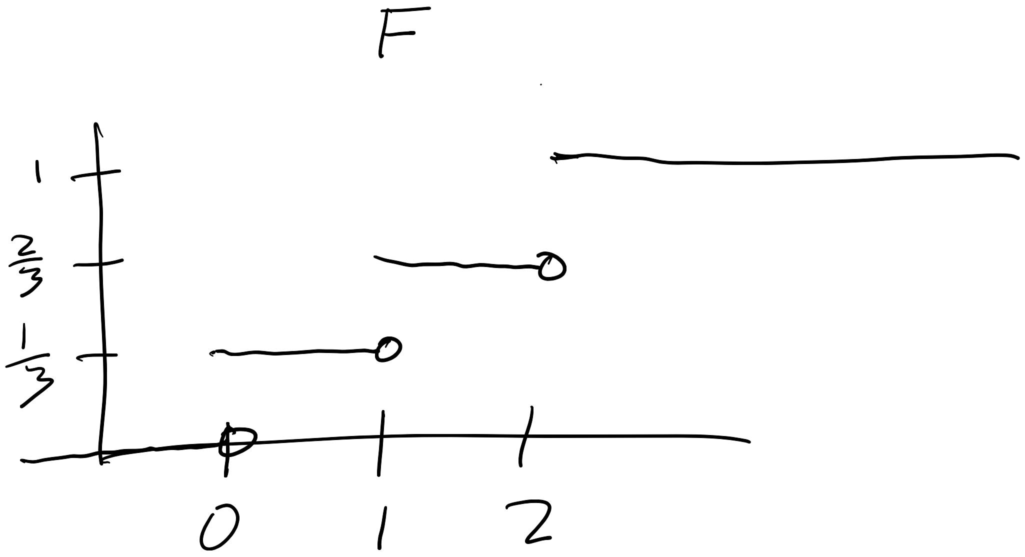
F



$P(X=1) = \frac{1}{6}$

$P(X=2) = \frac{1}{2} - \frac{1}{6}$

$P(X=3) = 1 - \frac{1}{2}$



$P(X=0) = \frac{1}{3}$

$P(X=1) = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$

$P(X=2) = \frac{1}{3}$