Section 2.5

Further topics

- won't cover hypergeometric r.v.'s

We cover **conditional independence**

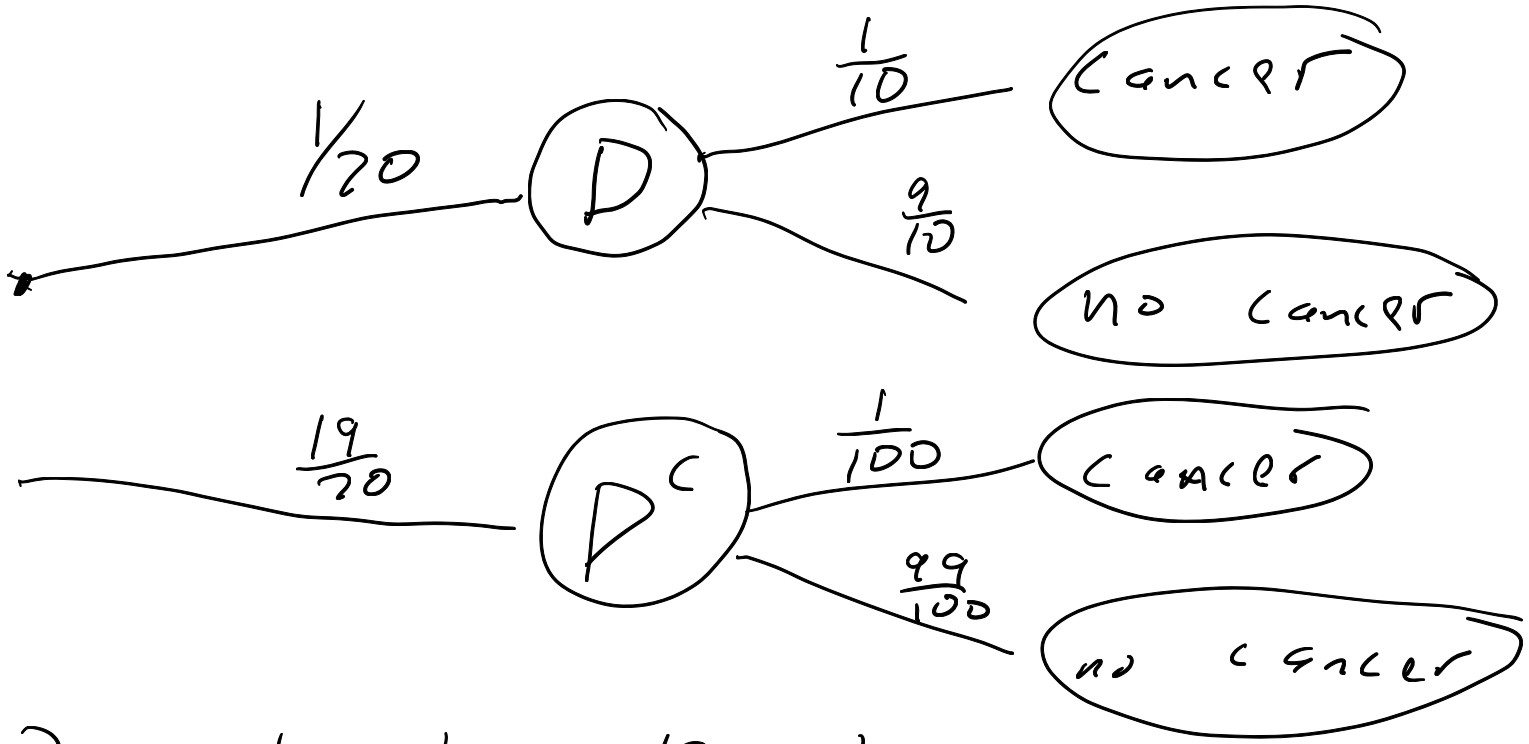
Ex) In general population, women with BRCA 1 gene are more prone to breast cancer.

Model (fabricated, but illustrates the idea):

- Breast cancer afflicts 10% of women with the gene and afflicts 1% of women without the gene
- Gene occurs in 5% of women and is always passed on to children
- No other gene is associated with breast cancer

Q) You (female) have two daughters. Let A be the event that 1st gets breast cancer. Let B be the event that 2nd gets breast cancer. Are A and B independent? $P(A) = ?, P(B) = ?, P(AB) = ?$

$D = \{\text{mother has BRCA 1}\}$



$$P(A) = \underbrace{\frac{1}{20}\cdot\frac{1}{10}}_{P(D)\cdot P(A|D)} + \underbrace{\frac{19}{20}\cdot\frac{1}{100}}_{P(D^c)\cdot P(A|D^c)} \approx \frac{3}{200}$$

$$P(AB) = \underbrace{\frac{1}{20} \cdot \frac{1}{10} \cdot \frac{1}{10}}_{P(D) \cdot P(A|D) \cdot P(B|D)} + \underbrace{\frac{19}{20} \cdot \frac{1}{100} \cdot \frac{1}{100}}_{P(D^c) \cdot P(A|D^c) \cdot P(B|D^c)}$$

$$\approx \frac{1}{2000} \gg \underbrace{P(A) \cdot P(B)}_{\left(\frac{3}{200}\right)^2}$$

Dependent!

Conditional independence

Def: Events A and B are said to be *conditionally independent* given event D if

$$P(AB|D) = P(A|D) \cdot P(B|D)$$

- Assumes $P(D) \neq 0$
- Can be extended to multiple conditionally independent events