Section 2.2

Bayes' Formula

Again: Flip a coin three times. For each head, roll a die. Then sum the die rolls.

$$P(\, sum = 3 \mid 1\, H\,) = \underline{\qquad}$$

$$P(1\, H | sum = 3) = ?$$

Bayes' Formula:

$$P(\,B\mid A\,) = \frac{P(A\,|B)\cdot P(B)}{P(A)} = \frac{P(A\,|B)\cdot P(B)}{P(A\,|B)\cdot P(B) + P(A\,|B^C)\cdot P(B^C)}$$

So $P(1\,H|sum = 3) = \frac{P(sum=3\,|1\,H)\cdot P(1\,H)}{P(sum=3)} = \frac{\cdot}{P(sum=3)}$

Proof of Bayes' formula:

Rare disease paradox

Ex) Suppose a rare disease effects $1/10^6$ of the population. You randomly decide to get tested, and you test positive! What is the probability you have the disease?

$$D = \{you\ have\ disease\}$$
$$T = \{you\ test\ positive\}$$

The test is very accurate:

- Proportion of false positives = $\frac{1}{10^4} = P(T|D^c)$
- Proportion of false negatives = $\frac{1}{10^5} = P(\_\_\_\_\_)$

$$P(you\ have\ disease\ |you\ test\ positive) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Monty Hall Problem