

Chapter 2

Conditional probability and independence

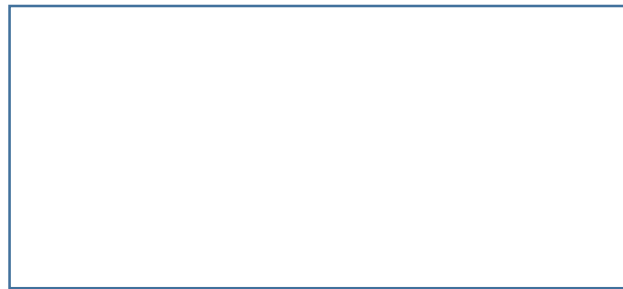
Section 2.1

Conditional probability

Ex) Let x be result of a die roll

Pmf: $p(k) = P\{x = k\} = \underline{\hspace{2cm}}$

Now I tell you " $x \leq 4$ ".



$P\{x = 5 \text{ given } x \leq 4\} = \underline{\hspace{1cm}}, P\{x = 6 \text{ given } x \leq 4\} = \underline{\hspace{1cm}}$

$P\{x = k \text{ given } x \leq 4\} = \underline{\hspace{1cm}}$ for $k = 1, 2, 3, 4$.

Conditioning on an event

Q: In general, we can *condition* on an event $B \in \Omega$. Conditioning updates all probabilities (new probability measure). What is the new probability measure?

We write $P(A | B)$ = “probability that A occurs given that B occurred”

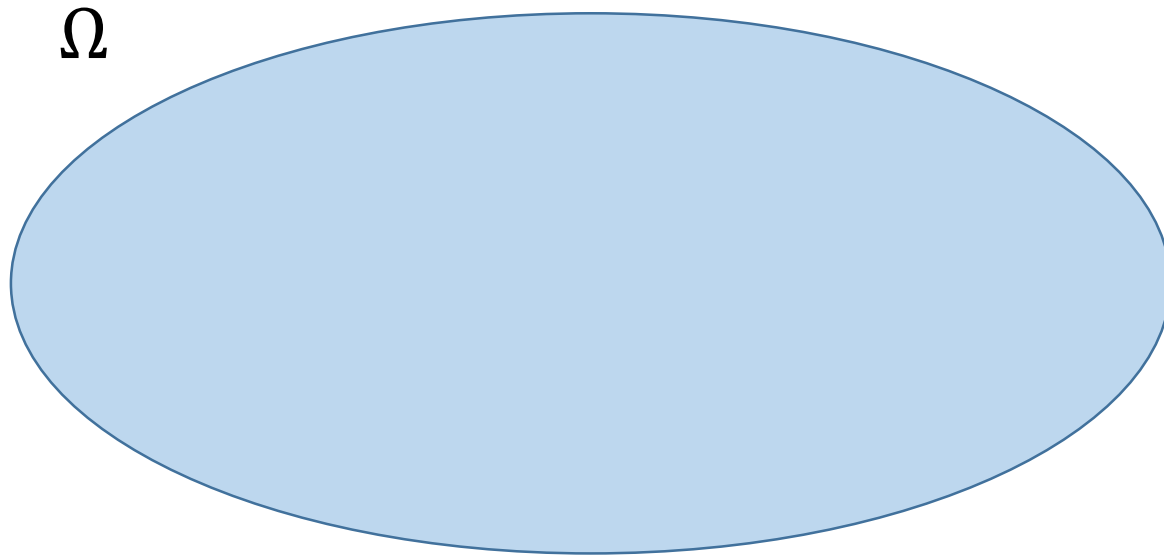
$$\begin{aligned} \text{So, } P(A \cap B) &= P(\text{"A and B occur"}) \\ &= P(\text{"B occurs"}) \cdot P(\text{"A occurs given B"}) \\ &= P(B) \cdot P(A | B) \end{aligned}$$

Def: Conditional probability

$$P(A | B) = \frac{P(A \cap B)}{P(B)} = \frac{P(AB)}{P(B)}$$

Alternative interpretation

When we condition on B , B becomes the new sample space. Divide by $P(B)$ to normalize, so that $P(B | B) = 1$. The collection $\{P(A | B)\}_{A \in \Omega}$ is an updated probability measure.



Ex) Roll two dice, $x = \text{sum}$.

$$P(x = 5 \mid \text{first die is } 3) = \underline{\hspace{15em}}$$

Ex) An urn has 9 red balls and 7 green balls. Draw 2 at random.

$$P\{\text{both green}\} = P(G_1 G_2) = \underline{\hspace{2cm}}$$

Now 4 drawn:

$$P(G_1 R_2 R_3 G_4) = \underline{\hspace{2cm}}$$

Fact:

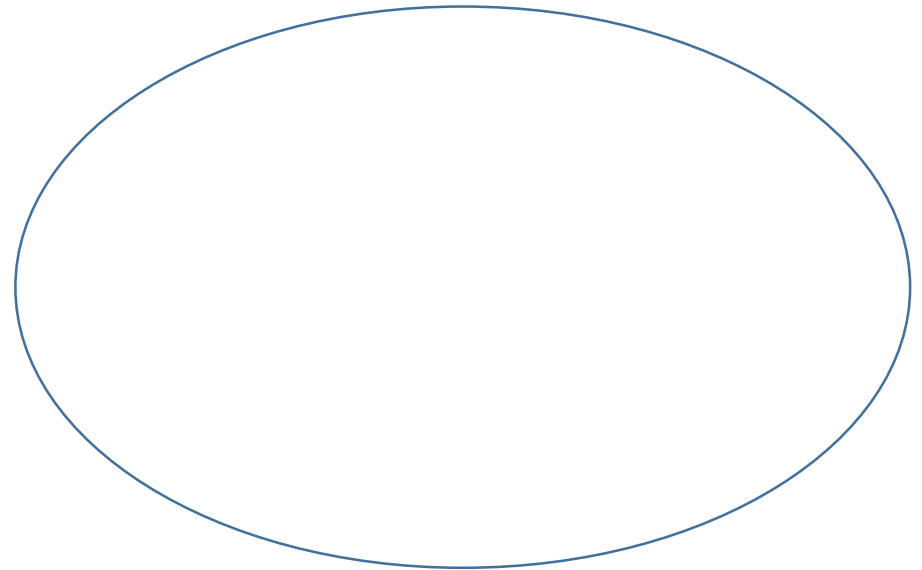
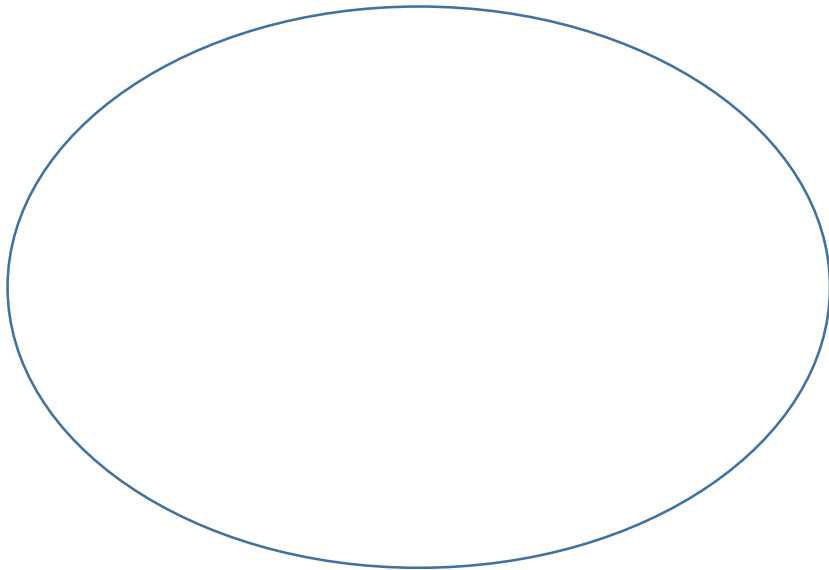
$$P(A_1 A_2 A_3 \cdots A_n) = P(A_1) \cdot P(A_2 | A_1) \cdot P(A_3 | A_1 A_2) \cdots P(A_n | A_1 A_2 \cdots A_{n-1})$$

Ex) An urn has 9 red balls and 7 green balls. Two drawn.

$$P(R_2) = \underline{\hspace{2cm}}$$

$$P(R_2) = P((G_1 \cup R_1) \cap R_2) =$$

$$\begin{aligned}\text{Fact: } P(A) &= P(AB) + P(AB^c) \\ &= P(A | B) \cdot P(B) + P(A | B^c) \cdot P(B^c)\end{aligned}$$



Fact: If B_1, B_2, \dots, B_n is a partition of Ω , then

$$P(A) = \sum_i P(A B_i) = \sum_i P(A | B_i) \cdot P(B_i)$$

Ex) Flip a coin three times. For each head, roll a die. What is $P\{\text{Sum of all dice rolled} = 3\}$?