Section 1.5

Random variables: a first look

Ex) Roll two dice, let x be the sum

x is a random variable. What is its relationship to the underlying sample space?

$$\Omega = \{(1,1), (1,2), (1,3), \ldots (6,5), (6,6)\}$$

x outputs a number for each outcome, i.e., x is a *function*. $x: \Omega \to \mathbb{R}$.

$$x\big((1,1)\big) = 2, \qquad x\big((3,5)\big) = \_\_\_\_, \qquad x\big((i,j)\big) = \_\_\_\_$$

Definition: A random variable (r.v.) is a function from Ω to $\mathbb{R}$.

Ex) Roll two dice, let x be the sum

What is $P\{x = 3\}$?

$\{x = 3\}$ is shorthand for $\{\omega \in \Omega: x(\omega) = 3\}$. This is the *preimage* of 3.

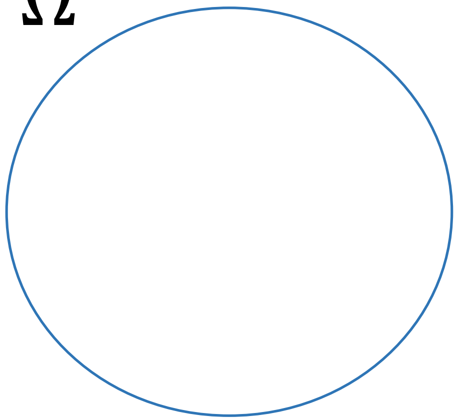
$$\{x = 3\} = \{(1,2), \quad \_\_\_\_\}$$

$$P\{x = 3\} = P\{\_\_\_\_\_\_\_\_\_\_\_\_\} = \_\_\_\_$$

Ex) Flip two coins. $x =$ number of heads

Which outcomes correspond with $x \leq 1$?

Ω



$$P\,\{x \leq 1\} = P\{\omega : x(\omega) \leq 1\} = P\{TT, \_\_\_\_, \_\_\_\_\} = \_\_\_\_$$

Probability distribution

Goal: completely specify the "randomness" of r.v. x.

Definition (def): The *probability distribution* of a r.v. x is the collection of probabilities $P\{x \in B\}$ for sets $B \subset \mathbb{R}$.

$$\{x \in B\} = preimage(B) = \{\omega \in \Omega : x(\omega) \in B\}$$

Discrete r.v.'s

Both random variables above were *discrete*, i.e, they take values on a discrete set, i.e., they only take at most a countably infinite number of values.

In discrete case, the probability distribution is captured by:

Def: The *probability mass function (pmf)* of a discrete r.v. x is the collection of probabilities $\mathrm{p}(k) = P\{x = k\}$ for all values k that x may take.

Relation to probability dist: $P(x \in B) =$ ________________

Ex) $x =$ number of heads in 2 flips

pmf of x? x can take the values: ________________

$p(0) =$ ____, $p(1) =$ ____, $p(2) =$ ____.

Q: Consider probability space defined by choosing an outcome uniformly at random on $[0,1]$. Can you make a discrete r.v. on this space?