

Section 1.4

Consequences of the rules of probability

First, some set theory notation:

Complement: $A^c = \{\text{everything not in } A\} = \{\omega \in \Omega: \omega \notin A\}$.

Union:

Intersection:

Difference:

Properties of probability measure

$$P(A) = 1 - P(A^c).$$

If $A = \bigcup_{i=1}^n A_i$ and A_i are pairwise disjoint, then

$$P(A) = \sum_i P(A_i).$$

If $B \subset A$, then $P(B) \leq P(A)$.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

Examples

Ex) Role 3 dice. What is the probability at least two match?

Ex) You have a biased coin that shows heads with probability p and tails with probability $1 - p$. Your friend and you take turns flipping, whoever gets heads first wins. Your friend goes first, but you choose p . What value should you choose?

Ex) You pick an integer uniformly at random from $1, 2, \dots, 100$. What is the probability that it is divisible by 10 or by 8?