Section 1.3

Infinitely many outcomes

Ex) Flip a coin until tails show. How many flips?

Ω = ____________________________

P{k flips} = P{1^{st} k-1 flips are heads} · P{k^{th} flip is tails}

= _________________ · _________ = _________

P{∞} =

Discrete sample spaces

Ω in the previous example was countably infinite. Finite and countably infinite sample spaces are called *discrete*.

Fact: When the sample space Ω is discrete,

$P(A) = \sum_{\omega \in A} P\{\omega\}$ for any event $A \subset \Omega$.

Proof: ___________________________

Uncountably infinite sample spaces

Ex) Pick number uniformly at random from $[0,1]$.

- $\Omega = [0,1]$
- P = ?

Special cases:

$P([0, 1/2]) =$ ___________

$P([0, 1/4]) =$ ___________

Can we write $P(A) = \sum_{\omega \in A} P\{\omega\}$?

Conclusion: In general, for uncountable Ω, $P(A) \neq \sum_{\omega \in A} P\{\omega\}$.

Q: But recall, P must satisfy

$$P(\Omega) = \sum_{i=1}^{\infty} P(A_i)$$

if A_1, A_2 … are pairwise disjoint, and $\cup_i A_i = \Omega$.

Is this a contradiction?