

Section 1.1

Sample spaces and probabilities

Sample space

Definitions illustrated with simple example: Roll two 4-sided dice

Definitions:

Sample point = possible outcome, usually denoted ω .

E.g., $\omega =$ _____

Sample space = the set of all sample points, denoted Ω

E.g, $\Omega = \{(1,1), (1,2), (1,3), (1,4), \text{_____,} \dots (3,4), (4,4)\}$

$\#\Omega$ = cardinality of Ω . E.g, $\#\Omega =$ _____

Definitions continued

Event := subset of Ω .

It can often be described with words.

E.g., $A = \{\text{The dice show the same number}\} = \{\underline{\hspace{10cm}}\}$

$F = \{\text{all possible events}\} = \text{all subsets of } \Omega$

$\#F = \underline{\hspace{2cm}}$

Probability measure: $P: F \rightarrow [0,1]$

- For event $A \in F$, $P(A) = \text{“probability Event } A \text{ occurs”}$
- In our example, $P(\{(1,3)\}) = P\{(1,3)\} = \underline{\hspace{1cm}} = P\{\omega\}$ for any $\omega \in \Omega$, and

$P\{(1,1), (2,2), (3,3), (4,4)\} = \underline{\hspace{10cm}}$

and $P(A) = \underline{\hspace{2cm}}$

Definitions continued

P satisfies:

- $0 \leq P(A) \leq \underline{\hspace{2cm}}$, for any $A \in F$
- $P(\emptyset) = \underline{\hspace{2cm}}$, $P(\Omega) = \underline{\hspace{2cm}}$
- If A_1, A_2, A_3, \dots are pairwise disjoint, $P(\bigcup_{i=1}^{\infty} A_i) = \underline{\hspace{2cm}}$

This implies $P(\bigcup_{i=1}^n A_i) = \underline{\hspace{2cm}}$

E.g., $P(\text{First die role equals 1 or 2}) = \underline{\hspace{20cm}}$

The triple, (Ω, F, P) , is called a *probability space*.

Q: Flip 3 coins. What is the corresponding probability space?

(Need to define Ω and P)