The University of British Columbia

Math 302 — Introduction to Probability

2019, February 13

Name: ______________________________ Student ID: ____________________

Instructions

- This exam consists of **4 questions** worth a total of 32 points.
- Make sure this exam has **4 pages** excluding this cover page.
- Note that there is a **table of discrete distributions** on Page 1, too.
- **Explain** your reasoning thoroughly, and **justify** all answers (even if the question does not specifically say so). No credit might be given for unsupported answers.
- All answers should be **simplified**.
- No calculators, notes, or other aids are allowed.
- If you need more space, use the back of the pages.
- Duration: **50** minutes.

Question	Points	Score
1	7	
2	10	
3	8	
4	7	
Total:	32	

Common Discrete Distributions

Random Variable X	$P(X = k)$	Mean	Variance
Ber(p)	$P(X = 0) = 1 - p$, $P(X = 1) = p$	p	$p(1 - p)$
Bin(n, p)	$\binom{n}{k} p^k (1 - p)^{n-k}$	np	$np(1 - p)$
Geom(p)	$p(1 - p)^{k-1}$	$\frac{1}{p}$	$\frac{1-p}{p^2}$

7 marks

1. You roll a 6-sided die and flip 6 fair coins. What is the probability that
   (a) You flip 2 Heads and roll a 2.
   (b) The number of Heads flipped is less than the number you rolled. Do NOT simplify.

10 marks

2. You're in love with person A. There is a 25% chance that A is also in love with you. If A is in love with you, A will smile at you every time you meet. If A is not in love with you, A will only smile at you 50% of the time.

   (a) A has smiled at you twice. What is the probability that A is in love with you?

   (b) A has smiled at you three times. What is the probability that A will smile at you also the next time you meet?

8 marks 3. Let X be a Geom(p) random variable. Compute $\mathbb{E}g(X)$ where

(a) $g(x) = x \cdot (x - 1)$

(b) $g(x) = x \cdot (x - 1) \cdot (x - 2)$.

7 marks

4. You are offered to play a game, where you have the opportunity to earn \$$X$, where X is a random variable with $\mathbb{E}X = 100$. The game is risky, so that X can also be negative, and you can afford to lose at most \$$N$.

   (a) Suppose that the variance $\sigma(X)^2 = 4000$. How large does N have to be so that you can be 90% sure that you will not lose more than you can afford?

   (b) Suppose you can afford to lose at most \$20. In what range does $\sigma(X)^2$ have to lie so that you can be 90% sure this will not happen?