Math 302, Assignment 5

1. Suppose X, Y are two discrete RV's with joint p.m.f. according to the table below.

   (a) Calculate the marginal p.m.f. of X and of Y.

   (b) Calculate $\mathbb{P}(0 < \sin(X) \cdot e^Y < 4)$.

   (c) Are X and Y independent?

   (d) Compute $\mathrm{cov}(X, Y)$.

Table 1: The joint p.m.f. of X, Y

$X \downarrow Y \rightarrow$	0	1	2	3
1/2	1/12	1/8	1/8	1/12
1	0	1/12	1/9	1/9
6	1/12	1/12	0	1/9

2. Let Z_1 and Z_2 be two points chosen uniformly from the unit disk, independently of each other. Let $d(Z_1, Z_2)$ be their Euclidean distance, that is, if $z_i = (x_i, y_i)$, then $d(z_1, z_2) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$. Compute $\mathbb{E}(d(Z_1, Z_2)^2)$.

3. Suppose that $X_1, \ldots, X_n$ are independent continuous random variables that all have the same c.d.f. $F(x)$. Define the random variable

$$Y = \max\{X_1, \ldots, X_n\}.$$

   Compute the c.d.f. and the p.d.f. of Y. Your answer should be in terms of $F(x)$. *Hint:* Express an inequality of the kind $\max\{X_1, \ldots, X_n\} \leq b$ in terms of separate inequalities for each X_i.

4. Let X and Y be two independent uniform random variables on $(0, 1)$.

   (a) Using the convolution formula, find the p.d.f. $f_Z(z)$ of the random variable $Z = X + Y$, and graph it.

   (b) What is the moment generating function of Z?

5. Textbook problems 8.2, 8.40, 8.43, 8.64, 9.2.