Math 302: Introduction to Probability

Instructor: Yaniv Plan

Teaching Assistant: Ali Fele Paranj

Class structure

- In-person lectures
- Weekly written homework due Fridays at 10pm (starting in Week 2)
- **Midterm June 3**, Final during final exam period

Grading Scheme:

- Homework 15% ← challenging, but easy to get good mark
- Midterm 35%
- Final 50%

} ← challenging.

Midterm Mark = max(Midterm, Final)

Textbook

"Introduction to Probability" by Anderson, Seppalainen, and Valko

Goal: Cover (almost) all sections not marked with  or 

Resources

- Piazza
- Textbook
- Lecture notes
- Written homeworks
- Office hours (starting in Week 2):
  - Professor OH: W 4-5pm location TBD
  - TA OH: Tu Th 12-1pm AUDX 130
- Practice problems on course website
  - Canvas quizzes (not marked)
  - Practice exams

Piazza

- Class message board (moderated)
- **Public posts**: Questions towards understanding mathematical content of the course.
- **Private posts to TA/Professor**: Questions/comments only the TA/Professor can answer.

Suggested study habits

- Read the corresponding part of the textbook before lecture
- Clarify concepts in lecture
- Active learning _during_ lecture
- Do 1-2 problems after lecture

- Written homework:
  - Your homework should be original
  - Show work
  - Some test problems will be based on homework problems

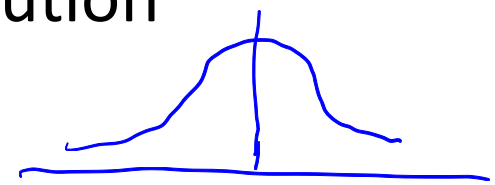
- Work out practice problems on Canvas

Why Probability?

- Foundational for Machine Learning and Data Science
  - Probabilistic modeling
  - Probabilistic algorithms
- Develop “probabilistic intuition” and critical thinking
- Rich theoretical area

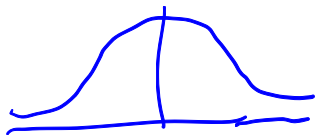
Highlights of this class

- Abstract and precise mathematical foundation
- Continuous and discrete random variables
- Functions of random variables
- Conditioning, expectation, variance
- Gaussian distribution



- Central limit theorem

$$\frac{1}{\sqrt{n}} \sum_{i=1}^{n} \pm 1 \approx$$



Next: A few questions related to "probabilistic intuition"

- Symmetry
- Paradoxes
- Real-life probabilistic thinking

Symmetry in random orderings

You have a deck of 52 randomly ordered cards. You draw the cards one by one.

1. What is the probability that the **first** card you draw is the ace of spades?

$1/52$

2. What is the probability that the **second** card you draw is the ace of spades?

$1/52$

3. What is the probability that the **37th** card you draw is the ace of spades?

$1/52$

More symmetry

Suppose each student in class needs a number from 1-100.

Option 1: Students line up according to last name and pick a random number from a bag (and keep it).

Option 2: Students line up randomly and are given numbers in order.

In which case is it more likely that you and your two friends get the numbers 1, 2, and 3?

Ans: Same, in both cases numbers are given uniformly at random

Birthday Problem

Q2: In a 100 student class, what is the probability that at least 2 students share a birthday?

Ans: $P(\text{at least 2 share}) = 1 - P(\text{none share})$

$P(\text{none share}) = ?$

Experiment: Line up students, each student says their bday and says "no" if it doesn't match a prev. bday

prob: $\frac{365}{365}$ $\frac{364}{365}$ $\frac{365}{365}$ ... $\frac{266}{365}$

$$P(\text{none share}) = \frac{365}{365} \cdot \frac{364}{365} \cdots \frac{266}{365} \approx \frac{1}{10^6}$$

$$P(\text{at least 2 share}) \approx 1 - \frac{1}{10^6} = 99.9999\%$$

Sally Clark Case (true story)

- Sally had 2 kids, both victims of sudden infant death syndrome (SIDS) (i.e., both children died in their sleep for unexplained causes)
- 1998: She is charged with murder
- Prosecution: SIDS occurs in $\frac{1}{8500}$ infants in general thus

P(2 SIDS) = $\frac{1}{8500} \cdot \frac{1}{8500} \approx \frac{1}{70 \text{ million}}$ Absurd!

assumes independence of SIDS

Perhaps jury thinks: P(Innocence) $\not\approx$ $\frac{1}{70 \text{ million}}$

$\neq$

Verdict: guilty