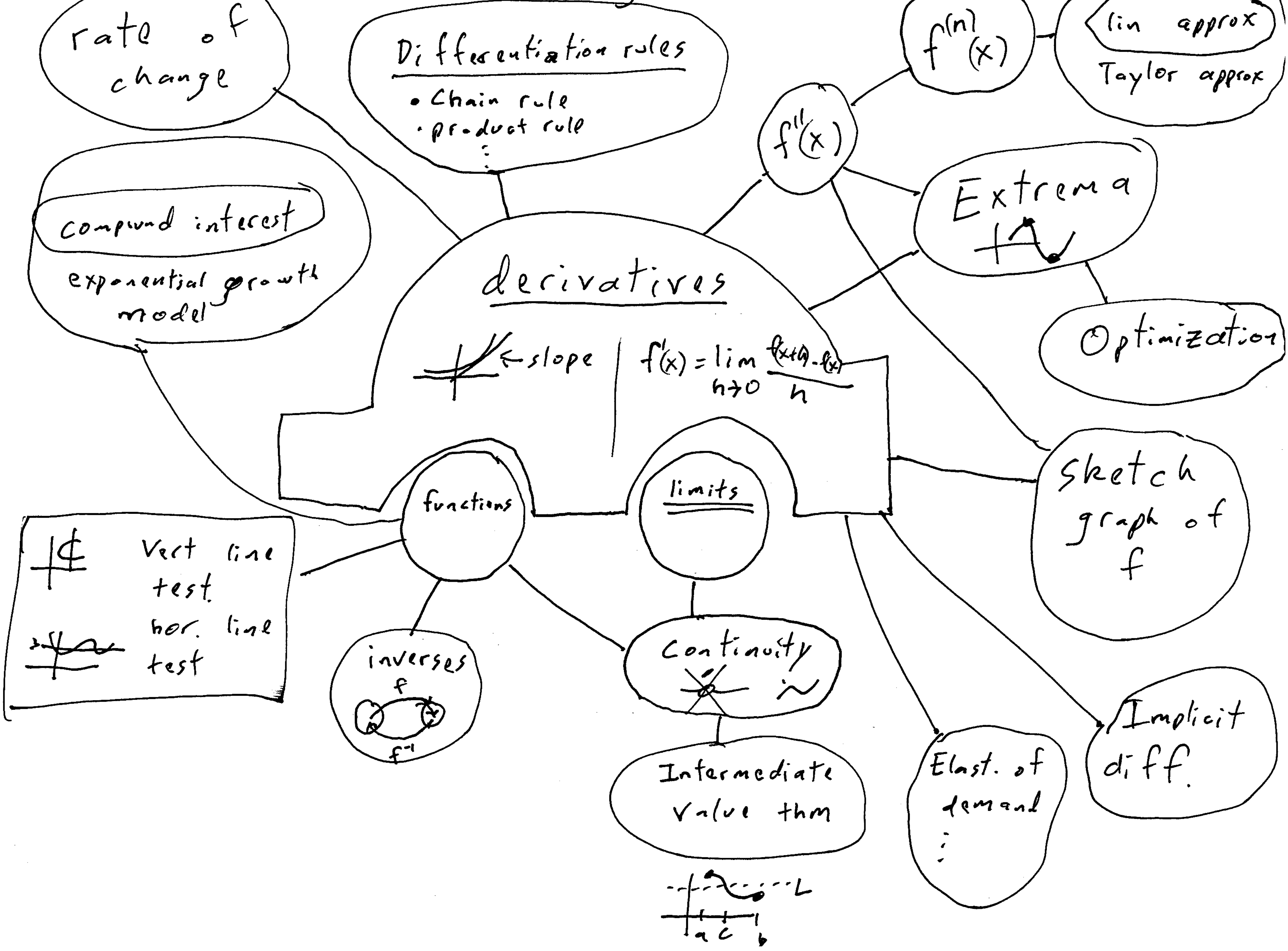
Concept graph



Questions

① Suppose $f'(3) = 5$, $f(3) = 2$.

Find $\lim_{x \to 3} \dfrac{f(x) \cdot (x-3)^2}{(f(x) - 2)^2} + \dfrac{1}{f(x)}$

② Simplify $\ln\left(x^{\ln(\cos^3(\ln(x) \cdot x))}\right)$

③ Let $f(x) = \cos\left(e^{\arccos(x^3)}\right)$. Find $f'(x)$.

④ Graph $f(x) = 3x^{\frac{1}{4}} - x^{\frac{1}{2}} - 2$

⑤ Find the 3rd order Taylor polynomial for $f(x) = \frac{x}{1-x}$ centered at $x = 2$.

① Note: f is cont. at $x=3$.

$$\Rightarrow \lim_{x\to 3} f(x) = f(3) = 2$$

Also, $$\lim_{x\to 3} \frac{f(x)-f(3)}{x-3} = \lim_{x\to 3} \frac{f(x)-2}{x-3} = f'(3) = 5$$

$$\frac{a\cdot b}{c} = \frac{a}{\frac{c}{b}}$$

$$\lim_{x\to 3} \frac{f(x)\,(x-3)^2}{(f(x)-2)^2} + \frac{1}{f(x)} = \lim_{x\to 3} \frac{f(x)}{\frac{(f(x)-2)^2}{(x-3)^2}}$$

$$= \lim_{x\to 3} f(x) \cdot \frac{1}{\lim_{x\to 3} \frac{(f(x)-2)^2}{(x-3)^2}} + \frac{1}{\lim_{x\to 3} f(x)}$$

$$= 2 \cdot \frac{1}{\left(\lim_{x\to 3} \frac{f(x)-2}{x-3}\right)^2} + \frac{1}{2}$$

$$= 2 \cdot \frac{1}{(f'(3))^2} + \frac{1}{2} = \boxed{2 \cdot \frac{1}{25} + \frac{1}{2}}$$

② $$\ln\left(x^{\ln(\cos^3(\ln(x)\cdot x))}\right)$$

$$\ln(a^b) = b\ln(a)$$

$$= \ln(\cos^3(\ln(x)\cdot x)) \cdot \ln(x)$$

$$= 3\ln(\cos(\ln(x)\cdot x)) \cdot \ln(x)$$

③ $$f(x) = \cos\left(e^{\arccos(x^3)}\right)$$

$$f'(x) = -\sin\left(e^{\arccos(x^3)}\right) \cdot \frac{d}{dx} e^{\arccos(x^3)}$$

$$= -\sin\left(e^{\arccos(x^3)}\right) \cdot e^{\arccos(x^3)} \cdot \frac{d}{dx}\arccos(x^3)$$

$$= \boxed{-\sin\left(e^{\arccos(x^3)}\right) \cdot e^{\arccos(x^3)} \cdot \frac{-1}{\sqrt{1-(x^3)^2}} \cdot 3x^2}$$

(4) $f(x) = 3x^{\frac{1}{4}} - x^{\frac{1}{2}} - 2$

- Domain: $x \geq 0$ ie. $[0, \infty)$
- Asymptotes: No vert

$\lim_{x \to \infty} f(x) = -\infty$ $\quad$ ($-x^{\frac{1}{2}}$ dominates)

- crit pts, sign f'.

$$f'(x) = \frac{3}{4} x^{-\frac{3}{4}} - \frac{1}{2} x^{-\frac{1}{2}}$$

$$= \frac{1}{4} \cdot x^{-\frac{3}{4}} \left(3 - 2 \cdot x^{\frac{3}{4}} \cdot x^{-\frac{1}{2}}\right)$$

$$= \frac{x^{-\frac{3}{4}}}{4} \left(3 - 2x^{\frac{1}{4}}\right)$$

Option 2:

Crit pt: $f'(x) = 0$

$$\Rightarrow \frac{3}{4} x^{-\frac{3}{4}} - \frac{1}{2} x^{-\frac{1}{2}} = 0$$

$$\Rightarrow \frac{3}{4} x^{-\frac{3}{4}} = \frac{1}{2} x^{-\frac{1}{2}}$$

$$\Rightarrow \frac{3}{4} x^{-\frac{3}{4}} \cdot x^{\frac{1}{2}} = \frac{1}{2}$$

$$\Rightarrow \frac{3}{4} x^{-\frac{1}{4}} = \frac{1}{2}$$

$\Rightarrow \frac{3}{2} = x^{\frac{1}{4}} \Rightarrow x = \left(\frac{3}{2}\right)^4$ is a crit pt

Note $x=0$ is not a crit pt because it is on boundary of domain.

$$\operatorname{sign}(f'(x)) = \begin{cases} + & x<\left(\frac{3}{2}\right)^4 \quad \text{(plug in } x=1\text{)} \\ - & x>\left(\frac{3}{2}\right)^4 \quad \text{(plug in } x=16\text{)} \end{cases}$$

- Infl pts & sign(f''):

$$f''(x) = \frac{d}{dx} f'(x) = -\frac{9}{16} x^{-\frac{7}{4}} + \frac{1}{4} x^{-\frac{3}{2}}$$

Infl. pt: $f''(x)=0$

$$\Rightarrow -\frac{9}{16} x^{-\frac{7}{4}} + \frac{1}{4} x^{-\frac{3}{2}} = 0$$

multiply both sides by $16x^{\frac{7}{4}}$

$$\Rightarrow -9 + 4x^{\frac{7}{4}} \cdot x^{-\frac{6}{4}} = 0$$

$$\Rightarrow 4 \cdot x^{\frac{1}{4}} = 9$$

$$\Rightarrow x^{\frac{1}{4}} = \frac{9}{4}$$

$$\Rightarrow x = \left(\frac{9}{4}\right)^4$$

$$\operatorname{sign}(f'') = \begin{cases} - & x<\left(\frac{9}{4}\right)^4 \quad \text{Plug in } x=1 \\ + & x>\left(\frac{9}{4}\right)^4 \quad \text{Plug in } x=10000 \end{cases}$$

~~Plug in x=16~~

• Tabulate

x	0		$\left(\frac{3}{2}\right)^4$		$\left(\frac{9}{4}\right)^4$		∞
f'		$+$		$-$		$-$	
f''		$-$		$-$		$+$	
f			hor tan line		infl pt		

• Extra step: special pt $f(0) = -2$

Graph: