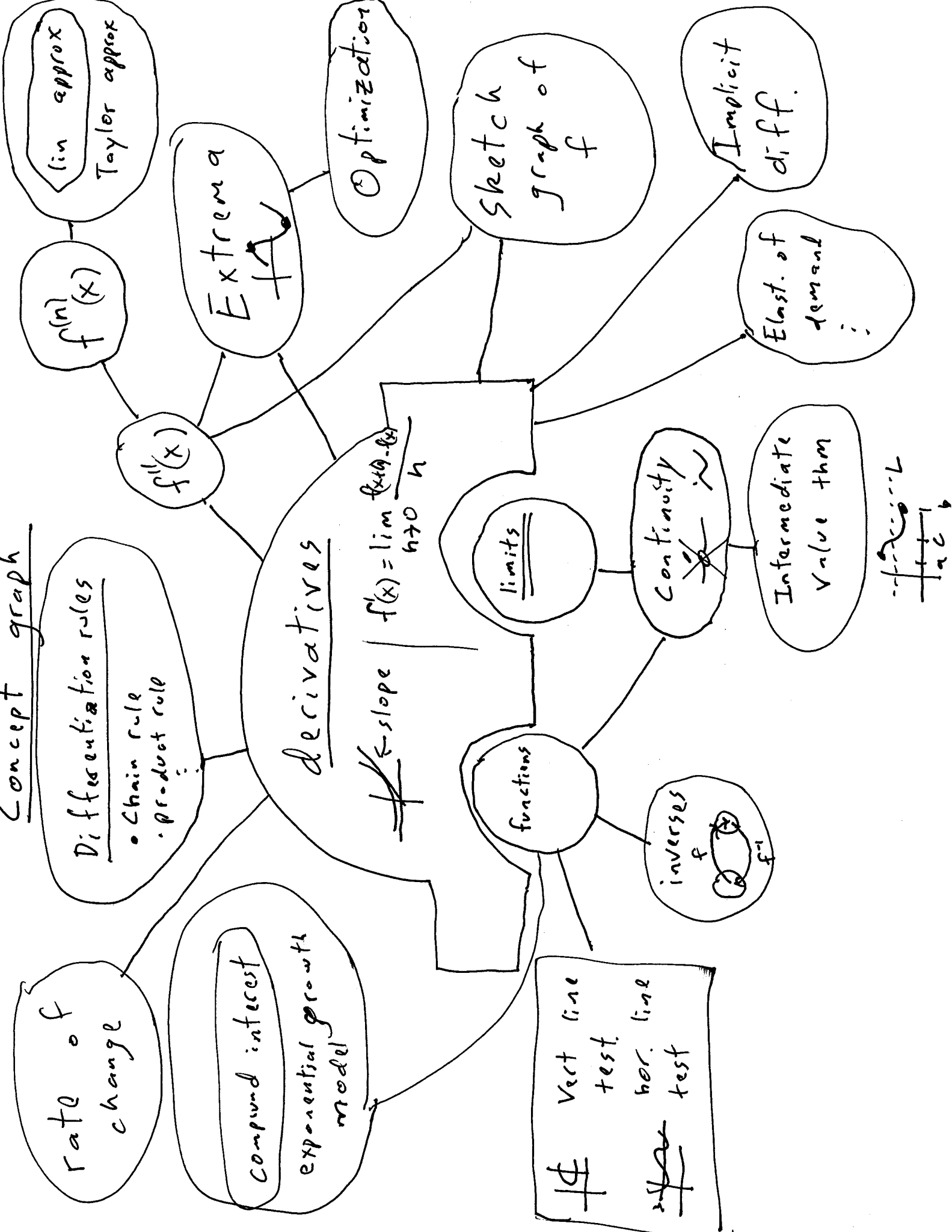


Concept graph



Differentiation rules

- Chain rule
- Product rule

Extrema

$f^{(n)}(x)$

$f'(x)$

Rate of change

Compound interest exponential growth model

derivatives

~~← slope~~ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

limits

~~Continuity~~

Intermediate Value thm

Implicit diff.

functions

inverses f f^{-1}

~~Vert line test~~
~~Hor. line test~~

Elast. of demand

~~f~~
 ~~f^{-1}~~

Questions

① Suppose $f'(3) = 5$, $f(3) = 2$.

Find $\lim_{x \rightarrow 3} \frac{f(x) \cdot (x-3)^2}{(f(x)-2)^2} + \frac{1}{f(x)}$

② Simplify $\ln(x^{\ln(\cos^3(\ln(x)-x))})$

③ Let $f(x) = \cos(e^{\arccos(x^3)})$. Find $f'(x)$.

④ Graph $f(x) = 3x^{\frac{1}{4}} - x^{\frac{1}{2}} - 2$

⑤ Find the 3rd order Taylor polynomial for $f(x) = \frac{x}{1-x}$ centered at $x=2$.

① Note: f is cont. at $x=3$.

$$\Rightarrow \lim_{x \rightarrow 3} f(x) = f(3) = 2$$

$$\text{Also, } \lim_{x \rightarrow 3} \frac{f(x) - f(3)}{x - 3} = \lim_{x \rightarrow 3} \frac{f(x) - 2}{x - 3} = f'(3) = 5$$

$$\lim_{x \rightarrow 3} \frac{f(x)(x-3)^2}{(f(x)-2)^2} + \frac{1}{f(x)} = \lim_{x \rightarrow 3} \frac{f(x)}{\frac{(f(x)-2)^2}{(x-3)^2}} + \frac{1}{f(x)}$$

$$= \lim_{x \rightarrow 3} f(x) \cdot \frac{1}{\lim_{x \rightarrow 3} \frac{(f(x)-2)^2}{(x-3)^2}} + \frac{1}{\lim_{x \rightarrow 3} f(x)}$$

$$= 2 \cdot \frac{1}{\left(\lim_{x \rightarrow 3} \frac{f(x)-2}{x-3}\right)^2} + \frac{1}{2}$$

$$= 2 \cdot \frac{1}{(f'(3))^2} + \frac{1}{2} = \boxed{2 \cdot \frac{1}{25} + \frac{1}{2}}$$

$$\frac{a \cdot b}{c} = \frac{a}{\frac{c}{b}}$$

② $\ln(x \ln(\cos^3(\ln(x) \cdot x)))$
 $= \ln(\cos^3(\ln(x) \cdot x)) \cdot \ln(x)$
 $= 3 \ln(\cos(\ln(x) \cdot x)) \cdot \ln(x)$

$$\ln(a^b) = b \ln(a)$$

③ $f(x) = \cos(e^{\arccos(x^3)})$

$$f'(x) = -\sin(e^{\arccos(x^3)}) \cdot \frac{d}{dx} e^{\arccos(x^3)}$$

$$= -\sin(e^{\arccos(x^3)}) \cdot e^{\arccos(x^3)} \cdot \frac{d}{dx} \arccos(x^3)$$

$$= \boxed{-\sin(e^{\arccos(x^3)}) \cdot e^{\arccos(x^3)} \cdot \frac{1}{\sqrt{1-(x^3)^2}} \cdot 3x^2}$$

fid

$$(4) \quad f(x) = 3x^{\frac{1}{4}} - x^{\frac{1}{2}} - 2$$

• Domain: $x \geq 0$ i.e. $[0, \infty)$

• Asymptotes: No vert

$$\lim_{x \rightarrow \infty} f(x) = -\infty \quad (-x^{\frac{1}{2}} \text{ dominates})$$

• Crit pts, sign f' .

$$f'(x) = \frac{3}{4}x^{-\frac{3}{4}} - \frac{1}{2}x^{-\frac{1}{2}}$$

$$= \frac{1}{4}x^{-\frac{3}{4}} \left(3 - 2 \cdot x^{\frac{3}{4}} \cdot x^{-\frac{1}{2}} \right)$$

$$= \frac{x^{-\frac{3}{4}}}{4} \left(3 - 2x^{\frac{1}{4}} \right)$$

Option 2:

$$\text{crit pt: } f'(x) = 0$$

$$\Rightarrow \frac{3}{4}x^{-\frac{3}{4}} - \frac{1}{2}x^{-\frac{1}{2}} = 0$$

$$\Rightarrow \frac{3}{4}x^{-\frac{3}{4}} = \frac{1}{2}x^{-\frac{1}{2}}$$

$$\Rightarrow \frac{3}{4}x^{-\frac{3}{4}} \cdot x^{\frac{1}{2}} = \frac{1}{2}$$

$$\Rightarrow \frac{3}{4}x^{-\frac{1}{4}} = \frac{1}{2}$$

$$\Rightarrow \frac{3}{2} = x^{\frac{1}{4}} \Rightarrow x = \left(\frac{3}{2}\right)^4$$

is a crit pt

~~(2)~~

Note $x=0$ is not a cost pt because it is on boundary of domain.

$$\text{sign}(f'(x)) = \begin{cases} + & x < \left(\frac{3}{2}\right)^4 & \text{(plug in } x=1) \\ - & x > \left(\frac{3}{2}\right)^4 & \text{(plug in } x=16) \end{cases}$$

• Infl pts & sign(f''):

$$f''(x) = \frac{d}{dx} f'(x) = -\frac{9}{16} x^{-\frac{7}{4}} + \frac{1}{4} x^{-\frac{3}{2}}$$

$$\text{Infl. pt: } f''(x) = 0$$

$$\Rightarrow -\frac{9}{16} x^{-\frac{7}{4}} + \frac{1}{4} x^{-\frac{3}{2}} = 0$$

Multiply both sides by $16x^{\frac{7}{4}}$

~~$$\Rightarrow -9 + 4x^{\frac{7}{4}} \cdot x^{-\frac{6}{4}} = 0$$~~

$$\Rightarrow 4 \cdot x^{\frac{1}{4}} = 9$$

$$\Rightarrow x^{\frac{1}{4}} = \frac{9}{4}$$

$$\Rightarrow x = \left(\frac{9}{4}\right)^4$$

$$\text{sign}(f'') = \begin{cases} - & x < \left(\frac{9}{4}\right)^4 \\ + & x > \left(\frac{9}{4}\right)^4 \end{cases}$$

Plug in $x=1$

~~Plug in $x=16$~~

Plug in $x=10000$

(3)

• Tabulate

x	f'	f''	f
0	$+$	$-$	\cap
$(\frac{3}{2})^4$			hor tan line
	$-$	$-$	\cap
$(\frac{9}{4})^4$			infl. pt
	$-$	$+$	\cup
∞			

• Extra step: special pt $f(0) = -2$

Graph:

