Ex) Find absolute extrema of $f(x) = x^4 - 2x^3$ on interval $[-2, 2]$:

1. Locate crit. pts in $(-2, 2)$:

$$f'(x) = 4x^3 - 6x^2$$

Always defined.

Check where $f'(x) = 0$.

Set $0 = f'(x) = 4x^3 - 6x^2 = 2x^2(2x-3)$

$\Rightarrow \boxed{0, \frac{3}{2}}$ are crit pts

2. Evaluate f at the crit pts and end pts $\{0, \frac{3}{2}, -2, 2\}$:

a) $f(0) = 0$

b) $f(\frac{3}{2}) = (\frac{3}{2})^4 - 2(\frac{3}{2})^3 = \frac{81}{16} - \frac{2 \cdot 27}{8} = \frac{81}{16} - \frac{4 \cdot 27}{16} = -\frac{27}{16}$

c) $f(-2) = (-2)^4 - 2 \cdot (-2)^3 = 32$

d) $f(2) = 2^4 - 2 \cdot 2^3 = 0$

3. Choose largest and smallest values from $\{0, -\frac{27}{16}, 32, 0\}$

Absolute max $= 32 = \cancel{f(\frac{3}{2})} = f(-2)$

Absolute min $= -\frac{27}{16} = f(\frac{3}{2})$

Ex) Find absolute extrema of $x^{2/3}(2-x)$ on $[-1,2]$:

1. Locate crit. pts in $(-1,2)$:

$$f'(x) = x^{2/3}\overset{(-1)}{\cancel{(2-x)}} + \frac{2}{3}x^{-\frac{1}{3}}(2-x) = -x^{2/3} + \frac{2(2-x)}{3x^{1/3}}$$

Not defined at $x=0$.

$\Rightarrow 0$ is a crit pt.

Set $f'(x)=0$ to find other crit pts:

$$0 = -x^{2/3} + \frac{2(2-x)}{3x^{1/3}} = \frac{-3x^{1/3}\cdot x^{2/3} + 2(2-x)}{3x^{1/3}}$$

$$= \frac{-3x+4-2x}{3x^{1/3}} = \frac{-5x+4}{3x^{1/3}} = 0$$

$\Rightarrow x = \frac{4}{5}$ is a crit pt.

2. Evaluate f. at crit pts and end pts: $\{0, \frac{4}{5}, -1, 2\}$:

a) $f(0)=0$

b) $f\left(\frac{4}{5}\right) = \left(\frac{4}{5}\right)^{2/3}\left(2-\frac{4}{5}\right) = \left(\frac{4}{5}\right)^{2/3}\frac{6}{5}$

c) $f(-1) = (-1)^{2/3}(2--1) = 3$

d) $f(2) = 2^{2/3}(2-2) = 0$

3. Choose largest and smallest from

$\{0, \left(\frac{4}{5}\right)^{2/3} \cdot \frac{6}{5}, 3, 0\}$

Absolute max $= 3 = f(-1)$

Absolute min $= 0 = f(0) = f(2)$

Ex) Find absolute extrema of $e^{x^3 - x}$ on $[0, 2]$:

1. Locate crit pts.

$f'(x) = (3x^2 - 1)e^{x^3 - x}$

Always defined. Set $f' = 0$:

$$\frac{0}{e^{x^3-x}} = \frac{f'(x) = (3x^2-1)e^{x^3-x}}{e^{x^3-x}}$$

$\Rightarrow 0 = 3x^2 - 1$

$\Rightarrow x^2 = \frac{1}{3}$

$\Rightarrow x = \frac{1}{\sqrt{3}}, \cancel{-\frac{1}{\sqrt{3}}}$ Not in interval $(0, 2)$.

2. Evaluate f at crit pts & end pts: $\{\frac{1}{\sqrt{3}}, 0, 2\}$:

a) $f\left(\frac{1}{\sqrt{3}}\right) = e^{\left(\frac{1}{\sqrt{3}}\right)^3 - \frac{1}{\sqrt{3}}} = e^{\frac{1}{3}\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{3}}} = e^{\left(\frac{1}{3}-1\right)\frac{1}{\sqrt{3}}} = e^{-\frac{2}{3\sqrt{3}}}$

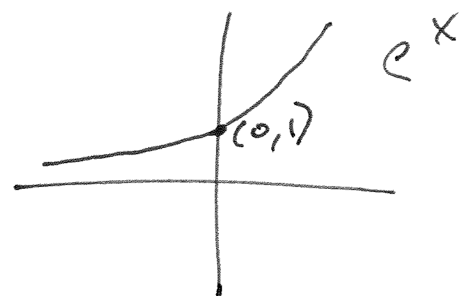
b) $f(0) = e^0 = 1$

c) $f(2) = e^{8-2} = e^6$

3. Choose largest smallest from $\{e^{-\frac{2}{3\sqrt{3}}}, 1, e^6\}$

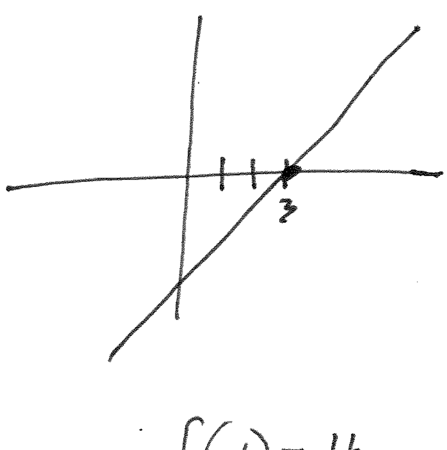
Absolute max $= e^6$

Absolute min $= e^{-\frac{2}{3\sqrt{3}}}$

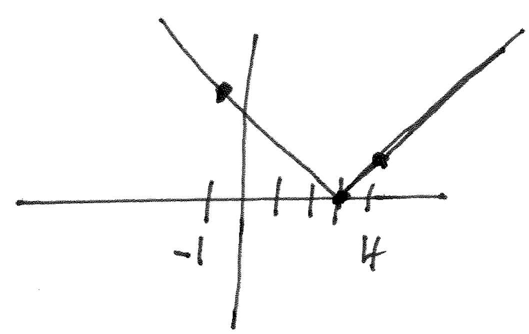


Ex) Find absolute min & max of $|x-3|$ on $[-1, 4]$:

Graph $y = x-3$:



Graph $y = |x-3|$

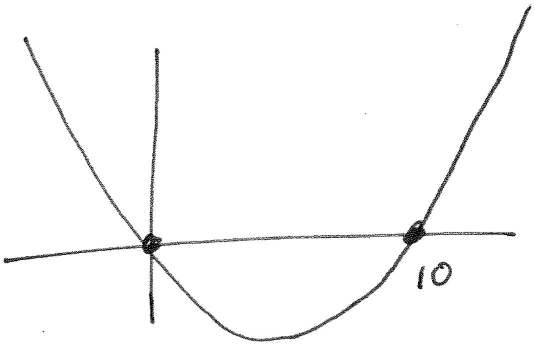


Absolute max $= f(-1) = 4$

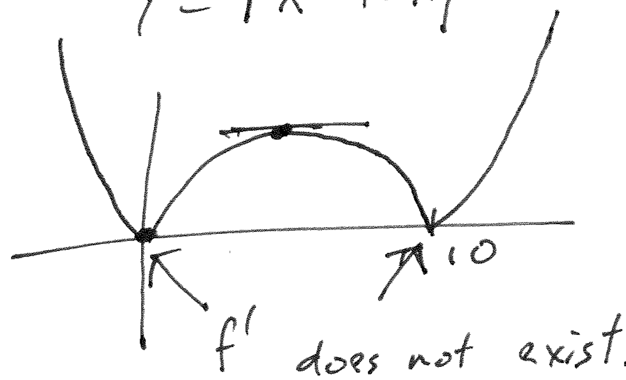
Absolute min $= f(3) = 0$

Ex) $f(x) = |x^2 - 10x|$ on interval $[0, 11]$:

sketch graph of
$y = x^2 - 10x = x(x-10)$



Sketch graph of
$y = |x^2 - 10x|$



1. Find crit pts.

   f' does not exist at $\overset{\text{on boundary}}{\downarrow}$ $0, 10$, thus 10 is a crit pt.

   set $f' = 0$ in $(0, 10)$ where $f(x) = -x^2 + 10x$

$0 = f'(x) = -2x + 10$

$\Rightarrow x = 5$ is a crit pt.

2. Evaluate f at crit pts, end pts $\{10, 5, 0, 11\}$

a) $f(10) = 0$

b) $f(5) = |25 - 50| = 25$

c) $f(0) = 0$

d) $f(11) = |121 - 110| = 11$

3. Absolute max $= 25 = f(5)$, Absolute min $= 0 = f(10) = f(0)$