

Price elasticity of demand

• Short version: "Demand elasticity"

Def Let P be price, q be quantity demanded. Then demand elasticity = $\epsilon := \frac{P}{q} \cdot \frac{dq}{dP} \approx \frac{\% \text{ change in quantity demanded}}{\% \text{ change in price}}$

• Note: $\epsilon < 0$ since $\frac{P}{q} > 0$ and $\frac{dq}{dP} < 0$.

What is it good for?

Let $R(P) = P \cdot q(P)$ be revenue as a function of price.

Then

$$\begin{aligned} MR = R'(P) &= \frac{d}{dP}(P \cdot q(P)) = P \cdot \frac{dq}{dP} + 1 \cdot q(P) \\ &= q(P) \left(1 + \frac{P}{q(P)} \cdot \frac{dq}{dP} \right) \\ &= q(P) (1 + \epsilon) \end{aligned}$$

How can we tell if $R'(P)$ is positive?

Note: $q(p)$ is always positive, and thus, the sign of $R'(p)$ is the sign of $1 + \epsilon$.

There are three cases:

① $\epsilon < -1 \Rightarrow R'(p) < 0$. Then we say the good is price elastic.

~~\Rightarrow Increase price p_2~~

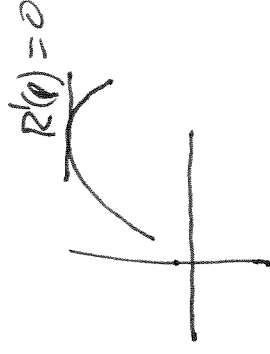
\Rightarrow Decrease price to increase revenue.

② $\epsilon > -1 \Rightarrow R'(p) > 0$. Then we say the good is price inelastic.

\Rightarrow Increase price to increase revenue.

③ $\epsilon = -1 \Rightarrow R'(p) = 0$. Then we say the good is price unit elastic.

\Rightarrow This price maximizes revenue.



Ex) Suppose demand curve for i-chalk is

$$5q + p = 1000$$

a) Compute price elasticity.

$$\epsilon = \frac{p}{q} \cdot \frac{dq}{dp}$$

Step 1: Solve for q in demand equation:

$$5q = 1000 - p \Rightarrow q = \frac{1000 - p}{5} \Rightarrow \frac{dq}{dp} = \frac{dp}{dp} \left(\frac{1000 - p}{5} \right) = -\frac{1}{5}$$

Step 2: Calculate

$$\epsilon = \frac{p}{\left(\frac{1000 - p}{5} \right)} \cdot \frac{dq}{dp} = \frac{p}{\left(\frac{1000 - p}{5} \right)} \left(-\frac{1}{5} \right) = \frac{5p}{1000 - p} \cdot \left(-\frac{1}{5} \right) = \left[\frac{-p}{1000 - p} \right]$$

b) What is price elasticity when price is \$100?

$$\epsilon = \frac{-100}{1000 - 100} = -\frac{1}{9}$$

c) ~~Price~~ Price elastic? Price inelastic? Price unit elastic?

Price inelastic

d) What is percent change in quantity demanded if price is 100 and increases by 3%? Use demand elasticity.

$$\text{Recall } E \approx \frac{\% \text{ change in quantity demanded}}{\% \text{ change in price}}$$

$$\text{Thus } -\frac{1}{3} = \frac{\% \text{ change in quantity demanded}}{3}$$

$$\Rightarrow \boxed{\% \text{ change in quantity demanded} = 3 \cdot -\frac{1}{3} = -1}$$

e) To increase revenue, what should the company do?
~~Decrease~~ Increase price since it is price inelastic.