Ex) $f(x) = \dfrac{(x^3-1)^4\sqrt{3x-1}}{x^2+4}$ $\qquad f'(x) = ?$

Step 1: take ln of both sides:

$$\ln(f(x)) = \ln\left(\frac{(x^3-1)^4\sqrt{3x-1}}{x^2+4}\right) = \ln\left((x^3-1)^4\right) + \ln\left(\sqrt{3x-1}\right) - \ln\left(x^2+4\right)$$

why? $\ln(ab) = \ln a + \ln b$

$\ln(abc) = \ln a + \ln b + \ln c$

$$\boxed{\ln\left(\frac{ab}{c}\right) = \ln a + \ln b - \ln c}$$

Thus, $\ln(f(x)) = 4\ln(x^3-1) + \frac{1}{2}\ln(3x-1) - \ln(x^2+4)$

Step 2: Take $\frac{d}{dx}$ of both sides:

$$\frac{d}{dx}\ln(f(x)) = \frac{d}{dx}4\ln(x^3-1) + \frac{d}{dx}\frac{1}{2}\ln(3x-1) - \frac{d}{dx}\ln(x^2+4)$$

$$\Rightarrow \frac{1}{f(x)}\cdot f'(x) = 4\frac{1}{x^3-1}\cdot 3x^2 + \frac{1}{2}\frac{1}{3x-1}\cdot 3 - \frac{1}{x^3+4}\cdot 2x$$

Thus, multiply both sides by $f(x)$ to give

$$f'(x) = f(x)\left[\frac{12x^2}{x^3-1} + \frac{3}{2(3x-1)} - \frac{2x}{x^2+4}\right]$$

$$\boxed{f'(x) = \frac{(x^3-1)^4\sqrt{3x-1}}{x^2+4}\left[\frac{12x^2}{x^3-1} + \frac{3}{2(3x-1)} - \frac{2x}{x^2+4}\right]}$$

Ex) $f(x) = (x+1)^{\ln x}$. What is $f'(x)$?

① Take ln of both sides:

$$\ln f(x) = \ln\left((x+1)^{\ln x}\right) = \ln(x)\cdot\ln(x+1)$$

② Take $\frac{d}{dx}$ of both sides:

$$\frac{d}{dx}\ln f(x) = \frac{d}{dx}\left(\ln(x)\cdot\ln(x+1)\right)$$

$$\frac{f'(x)}{f(x)} = \ln(x)\cdot\frac{1}{x+1}\cdot 1 + \frac{1}{x}\cdot\ln(x+1)$$

Multiply both sides by $f(x)$:

$$f'(x) = f(x)\left[\frac{\ln x}{x+1} + \frac{\ln(x+1)}{x}\right]$$

$$\boxed{f'(x) = (x+1)^{\ln x}\left[\frac{\ln x}{x+1} + \frac{\ln(x+1)}{x}\right]}$$

Ex) $f(x) = \dfrac{(x^3+1)^{\ln(x)}}{(\ln x)^{10}}$

① Take ln of both sides:

$$\ln(f(x)) = \ln\left(\frac{(x^3+1)^{\ln x}}{(\ln x)^{10}}\right) = \ln\left((x^3+1)^{\ln x}\right) - \ln\left((\ln(x))^{10}\right)$$

$$= \ln(x)\cdot\ln(x^3+1) - 10\ln(\ln(x))$$

② $\frac{d}{dx}$ both sides:

$$\frac{f'(x)}{f(x)} = \frac{d}{dx}\left(\ln(x)\cdot\ln(x^3+1) - 10\ln(\ln(x))\right)$$

$$= \ln(x)\frac{1}{x^3+1}\cdot 3x^2 + \frac{1}{x}\cdot\ln(x^3+1) - 10\frac{1}{\ln(x)}\cdot\frac{1}{x}$$

Multiply both sides by $f(x)$: $\boxed{f'(x) = \dfrac{(x^3+1)^{\ln x}}{(\ln x)^{10}}\left[\dfrac{\ln(x)\cdot 3x^2}{x^3+1} + \dfrac{\ln(x^3+1)}{x} - \dfrac{10}{x\ln x}\right]}$

③