Ex 1) $\frac{d}{dx} x^x = ?$

Try setting $y = x^x$ and take ln of both sides:

$\Rightarrow \ln y = \ln(x^x) = x \ln x$

Take $\frac{d}{dx}$ of both sides

$\Rightarrow \frac{d}{dx} \ln y = \frac{d}{dx} x \ln x$

$\Rightarrow \frac{1}{y} \cdot \frac{dy}{dx} = 1 \cdot \ln x + x \cdot \frac{1}{x}$ ← More steps

(1) $\frac{d}{dx} \ln y = \begin{pmatrix}\text{derivative of ln} \\ \text{evaluated at } y\end{pmatrix} \cdot \begin{pmatrix}\text{derivative} \\ \text{of } y\end{pmatrix}$

$= \frac{1}{y} \cdot \frac{dy}{dx}$

(2) $\frac{d}{dx}(x \ln x) = \frac{d}{dx}(x) \cdot \ln x + x \frac{d}{dx} \ln x$

$= 1 \cdot \ln x + x \cdot \frac{1}{x}$

$= \ln x + 1$

(1) = (2)

$\Rightarrow \frac{dy}{dx} \cdot \frac{1}{y} = \ln x + 1$

$\boxed{\frac{dy}{dx} = (\ln x + 1) \cdot y = (\ln x + 1) x^x}$

i.e. $\boxed{\frac{d}{dx} x^x = (\ln x + 1) x^x}$

Ex 2) $f(x) = \ln(1 + \ln(1 + \ln x))$

$$\frac{d}{dx} f(x) = \frac{1}{1 + \ln(1 + \ln x)} \cdot \frac{d}{dx}\left(1 + \ln(1 + \ln x)\right)$$

$$= \frac{1}{1 + \ln(1 + \ln x)} \cdot \left(0 + \frac{1}{1 + \ln x} \cdot \frac{d}{dx}(1 + \ln x)\right)$$

$$= \boxed{\frac{1}{1 + \ln(1 + \ln x)} \left(\frac{1}{1 + \ln x} \cdot \frac{1}{x}\right)}$$

Ex 3) Find the slope of the graph of $g(x) = \ln(f(2x))$ at $x = 1$, where $f(2) = 3$ & $f'(2) = 5$.

We want $g'(1)$.

$$g'(x) = \frac{d}{dx} \ln(f(2x)) = \frac{1}{f(2x)} \cdot \frac{d}{dx} f(2x) = \frac{1}{f(2x)} \cdot f'(2x) \cdot 2$$

$$g'(1) = \frac{1}{f(2)} \cdot f'(2) \cdot 2 = \frac{1}{3} \cdot 5 \cdot 2 = \boxed{\frac{10}{3}}$$

Ex) Down syndrome: Assume incidence of down syndrome fits ~~[illegible]~~ function

$$P(a) = \frac{1}{1{,}613{,}000} \cdot 1.2733^a$$ where a is age in years.

(1) At what age is the incidence of Down Syndrome equal to .01?

We need to solve $P(a) = .01$

$$\Rightarrow \frac{1}{1{,}613{,}000} 1.2733^a = .01$$

$$\Rightarrow 1.2733^a = .01 \cdot 1{,}613{,}000 = 16{,}130$$

Take ln of both sides:

$$\ln(1.2733^a) = a\ln(1.2733) = \ln(16{,}130)$$

$$\Rightarrow \boxed{a = \frac{\ln(16{,}130)}{\ln(1.2733)} \approx 40 \text{ yrs}}$$

(2) Compute $P'(a)$:

$$\frac{d}{da}\left(\frac{1}{1{,}613{,}000}\cdot 1.2733^{a}\right) = \frac{1}{1{,}613{,}000}\cdot 1.2733^{a}\cdot \ln(1.2733)$$

$$\Rightarrow \boxed{P'(a) \approx \frac{1}{6{,}676{,}000}\cdot 1.2733^{a}}$$

(3) Find $P'(35)$, $P'(46)$, interpret each.

$$P'(35) = \frac{1}{6{,}676{,}000}\cdot 1.2733^{35} \approx .0007$$

$$P'(46) = \frac{1}{6{,}676{,}000}\cdot 1.2733^{46} \approx .01$$

Roughly, if you are 35 and you wait another year to have a baby, the chance of Down syndrome increases by .0007.