Let $f(x) = \frac{x^2}{x^2+1}$. What is $f'(1)$?

Step 1: Definition of f':

$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

Step 2: Plug in f:

$$f'(1) = \lim_{h \to 0} \frac{\frac{(1+h)^2}{(1+h)^2+1} - \frac{1^2}{1^2+1}}{h} = \lim_{h \to 0} \frac{1+2h+h^2}{h(1+2h+h^2+1)} - \frac{1}{2h}$$

$$= \lim_{h \to 0} \frac{(1+2h+h^2)2 - 1\cdot(2+2h+h^2\cancel{+1})}{h(2+2h+h^2\cancel{+1})\cdot 2} = \lim_{h \to 0} \frac{\cancel{2}+4\cancel{h}+2h^{\cancel{2}}-\cancel{2}-2\cancel{h}-h^{\cancel{2}}}{\cancel{h}(2+2h+h^2)\cdot 2}$$

$$= \lim_{h \to 0} \frac{4+2h-2-h}{(2+2h+h^2)\cdot 2} = \frac{4-2}{2\cdot 2} = \frac{2}{4} = \boxed{\frac{1}{2}}$$

Let $f(x) = \sqrt{x+1}$. What is $f'(x)$?

<u>Step 1: Def of f':</u>

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

<u>Step 2: Plug in f:</u>

$$f'(x) = \lim_{h \to 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h} \cdot \frac{\sqrt{x+h+1} + \sqrt{x+1}}{\sqrt{x+h+1} + \sqrt{x+1}}$$

$$= \lim_{h \to 0} \frac{\cancel{x}+\cancel{h}^{\,1}+\cancel{1} - \cancel{(x+1)}}{\cancel{h}\left(\sqrt{x+h+1} + \sqrt{x+1}\right)}$$

$$= \frac{1}{\sqrt{x+1} + \sqrt{x+1}} = \boxed{\frac{1}{2\sqrt{x+1}}}$$