Assume $f(x), g(x)$ continuous at $x=3$, $g(3)=2$, &

$\lim_{x \to 3} x f(x) + g(x) = 1$.

Find $f(3)$.

Note $f(x)$ is cont. at $x=3$, thus $x f(x)$ is cont. at $x=3$ and so is $x f(x) + g(x)$. Since it is cont., we can plug in $x=3$ to the limit

$$\underbrace{\lim_{x \to 3} x f(x) + g(x)}_{= 1} = 3 \cdot f(3) + g(3) = 3 \cdot f(3) + 2$$

Thus, $3 f(3) + 2 = 1$

$\Rightarrow 3 f(3) = 1 - 2 = -1$

$\boxed{f(3) = -\frac{1}{3}}$

Show that the following equations have solutions:

1. $\underline{5^x - 10x - 7 = 0}$

$f(x) = 5^x - 10x - 7$

$$f(-1) = 5^{-1} - 10(-1) - 7 = \frac{1}{5} + 10 - 7 = 3\tfrac{1}{5} > 0$$

$$f(1) = 5^1 - 10 \cdot 1 - 7 = 5 - 10 - 7 = -12 < 0$$

Intermediate Thm says: There is some $c \in (-1, 1)$ so that $f(c) = 0$.

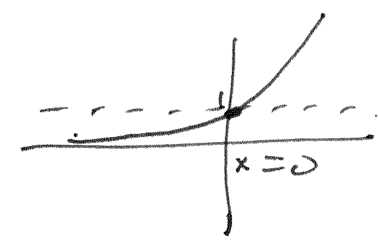
within

(2) $e^x = x^2$

$e^x - x^2 = 0$

set $f(x) = e^x - x^2$

~~For~~ We want to find a, b so that $f(a) \le 0$, $f(b) \ge 0$. We will use the following inequalities:



For $x \ge 0$, $e^x \ge 2^x$

Thus, $f(x) \ge 2^x - x^2$

$f(5) \ge 2^5 - 5^2 = 32 - 25 = 7$

For $x \le 0$, $e^x \le 1$

Thus, $f(x) \le 1 - x^2$

$f(-2) \le 1 - (-2)^2 = -3$

$\Rightarrow$ By interm. value thm, there is some $c \in (-2, 5)$ so that $f(c) = 0$.