Section 1.1

Sample spaces and probabilities

Sample space

Definitions illustrated with simple example: Roll two 4-sided dice

Definitions:

Sample point = possible outcome, usually denoted ω.

E.g., ω = (2, 3)

1st die is 2

2nd die is 3

Note: Curvy brackets (,) indicate an ordered list.

$(a,b) \neq (b,a)$

Sample space = the set of all sample points, denoted Ω

E.g, $\Omega = \{(1,1), (1,2), (1,3), (1,4),$ (2,1) $,\ldots(3,4), (4,4)\}$

Note: Squiggly brackets { } indicate unordered set.

$\{a,b\} = \{b,a\}$

$\#\Omega$ = cardinality of Ω. E.g, $\#\Omega$ = 16

Definitions continued

Event := subset of Ω.

It can often be described with words.

E.g., A = {The dice show the same number} = {(1,1), (2,2), (3,3), (4,4)}

$B = \{\text{sum of dice} = 3\} = \{(1,2), (2,1)\}$

F = {all possible events} = all subsets of Ω

power set

$\#F = 2^{\#\Omega}$

why? For an event A, you can circle every outcome in Ω which is in A, & cross out every outcome which is not... $\Rightarrow$ ... Thus, $\#F$ = # of sequences of O's and x's $= 2^{\#\Omega}$.

Probability measure: $P: F \to [0,1]$

- *For event* $A \in F$, $P(A)$ = *"probability Event A occurs"*
- In our example, $P(\{(1,3)\}) = P\{(1,3)\} = 1/16 = P\{\omega\}$ for any $\omega \in \Omega$, and

$P\{(1,1),(2,2),(3,3),(4,4)\} = \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{1}{4}$

and $P(A) = \frac{\#A}{\#\Omega} = \frac{\#A}{16}$

This holds whenever $\#\Omega < \infty$ & all outcomes are equally likely.

Definitions continued

$\emptyset = \{\ \} =$ empty set

P satisfies:

- $0 \leq P(A) \leq$ 1, for any $A \in F$
- $P(\emptyset) =$ 0, $P(\Omega) =$ 1
- If $A_1, A_2, A_3, \ldots$ are pairwise disjoint, $P(\cup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} P(A_i)$

pairwise disjoint →

This implies $P(\cup_{i=1}^{n} A_i) = \sum_{i=1}^{n} P(A_i)$

disjoint

For 4-sided die

E.g., $P(\text{First die role equals 1 or 2}) = P(\{\text{1st die} = 1\} \cup \{\text{1st die} = 2\})$

$= P\{\text{1st die} = 1\} + P\{\text{1st die} = 2\} = 1/2$

The triple, (Ω, F, P), is called a *probability space.*

Q: Flip 3 coins. What is the corresponding probability space?

(Need to define Ω and P)

$\Omega = \{H,T\}^{\otimes 3} = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$P\{HHH\} = 1/8$ (since all outcomes are equally likely)

$P(A) = \frac{\# A}{\# \Omega} = \frac{\# A}{8}$