*This midterm has **5 questions** on **8 pages**, for a total of 100 points.*

Duration: 50 minutes

- Read all the questions carefully before starting to work.
- Give complete arguments and explanations for all your calculations; answers without justifications will not be marked.
- Continue on the back of the previous page if you run out of space.
- Attempt to answer all questions for partial credit.
- This is a closed-book examination. **None of the following are allowed**: documents, cheat sheets or electronic devices of any kind (including calculators, cell phones, etc.)
- Smoking is bad for you.

Full Name (including all middle names): _____

Student-No: _____

Signature: _____

Question:	1	2	3	4	5	Total
Points:	36	16	16	16	16	100
Score:						

SHORT ANSWER QUESTIONS.

Please show your work and also underline your answer.
Each question is worth 6 marks, but an incorrect answer will be given at most 2 marks.
Unless otherwise stated, it is not necessary to simplify your answers.

6 marks 1. (a) Differentiate $g(x) = \dfrac{e^x}{x^e}$ with respect to x.

6 marks (b) Evaluate $\lim\limits_{n\to\infty} \left(\sqrt{4n^2+6} - 2n\right)$ or determine that it does not exist.

6 marks (c) Find $\frac{dy}{dt}$ when $y = \tan(\sin(t))$.

6 marks (d) What values of a and b makes the following function continuous and differentiable everywhere?

$$h(x) = \begin{cases} ax + b & \text{if } x < 5 \\ 4 - x & \text{if } x \geq 5 \end{cases}$$

6 marks (e) Find the *second* derivative of $f(\theta) = \sin(\theta)\cos(\theta)$.

6 marks (f) A somewhat contrived particle moves along the x-axis so that its position at time t is given by

$$x(t) = t + \sin(t)$$

For what values of t is the acceleration of the particle equal to zero?

FULL-SOLUTION PROBLEMS

In the remaining questions, justify your answers and **show all your work.** If you need more space, use the back of the *previous* page.

16 marks

2. Consider the following limit:

$$\lim_{x \to 4} \frac{\sqrt{x} - 2}{|4 - x|}.$$

Does this limit exist? If it does, carefully determine its value. If not, give a clear explanation of why it does not exist.

16 marks 3. Let $f(z) = \sqrt{z^2 + 2z}$. **Using the definition of derivative**, show that $f'(z) = \dfrac{z+1}{\sqrt{z^2+2z}}$. **No marks will be given for the use of differentiation rules**.

16 marks 4. Let f and g be differentiable functions that satisfy

$$f'(1) = g(0) = 1 \qquad \text{and} \qquad g(1) = g'(1) = 2$$

Now to be nasty, we define $h(x) = f\left(g(x^2) - x\right)$, but to be nice we assume that $h'(x)$ is continuous.

(a) What are $h'(0)$ and $h'(1)$?

(b) Does there exist a value a such that there is a horizontal tangent line to the curve $y = h(x)$ at $(a, h(a))$? Explain.

16 marks 5. How many tangent lines to the curve

$$y = \frac{x}{x-2}$$

pass through the point $(3, 3)$? Explain your answer.