This midterm has 5 questions on 8 pages, for a total of 100 points.

Duration: 50 minutes

- Read all the questions carefully before starting to work.
- Give complete arguments and explanations for all your calculations; answers without justifications will not be marked.
- Continue on the back of the previous page if you run out of space.
- Attempt to answer all questions for partial credit.
- This is a closed-book examination. None of the following are allowed: documents, cheat sheets or electronic devices of any kind (including calculators, cell phones, etc.)
- Smoking is bad for you.

Full Name (including all middle names): _______________________________

Student-No: _________________________________________________________

Signature: __________________________________________________________

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Score: ____________________________
SHORT ANSWER QUESTIONS.
Please show your work and also underline your answer.
Each question is worth 6 marks, but an incorrect answer will be given at most 2 marks.
Unless otherwise stated, it is not necessary to simplify your answers.

6 marks 1. (a) Differentiate \( g(x) = \frac{e^x}{xe} \) with respect to \( x \).

6 marks (b) Evaluate \( \lim_{n \to \infty} \left( \sqrt{4n^2 + 6} - 2n \right) \) or determine that it does not exist.
6 marks (c) Find $\frac{dy}{dt}$ when $y = \tan(\sin(t))$.

6 marks (d) What values of $a$ and $b$ makes the following function continuous and differentiable everywhere?

$$h(x) = \begin{cases} 
ax + b & \text{if } x < 5 \\
4 - x & \text{if } x \geq 5
\end{cases}$$
(e) Find the second derivative of \( f(\theta) = \sin(\theta) \cos(\theta) \).

(f) A somewhat contrived particle moves along the \( x \)-axis so that its position at time \( t \) is given by

\[
x(t) = t + \sin(t)
\]

For what values of \( t \) is the acceleration of the particle equal to zero?
FULL-SOLUTION PROBLEMS
In the remaining questions, justify your answers and show all your work. If you need more space, use the back of the previous page.

2. Consider the following limit:

\[
\lim_{x \to 4} \frac{\sqrt{x} - 2}{|4 - x|}.
\]

Does this limit exist? If it does, carefully determine its value. If not, give a clear explanation of why it does not exist.
3. Let \( f(z) = \sqrt{z^2 + 2z} \). Using the definition of derivative, show that \( f'(z) = \frac{z + 1}{\sqrt{z^2 + 2z}} \).

No marks will be given for the use of differentiation rules.
4. Let $f$ and $g$ be differentiable functions that satisfy

$$f'(1) = g(0) = 1 \quad \text{and} \quad g(1) = g'(1) = 2$$

Now to be nasty, we define $h(x) = f(g(x^2) - x)$, but to be nice we assume that $h'(x)$ is continuous.

(a) What are $h'(0)$ and $h'(1)$?

(b) Does there exist a value $a$ such that there is a horizontal tangent line to the curve $y = h(x)$ at $(a, h(a))$? Explain.
5. How many tangent lines to the curve

\[ y = \frac{x}{x - 2} \]

pass through the point (3, 3)? Explain your answer.