Midterm 2 practice problems

1. Let $\{N(t) : t \geq 0\}$ be a rate λ Poisson process.

   (a) Find the joint probability mass function of $N(t)$, $N(t+s)$ for some $s, t > 0$.

   (b) What is $P(N(1) = 1, N(2) = 2, N(3) = 3, N(4) = 4)$?

   (c) What is $E(T_1 + T_3 + T_5 | N(t) = 2)$?

   (d) Suppose that each event is Type I with probability p, Type II with probability $1 - p$. Let $N_1(t)$ be the number of Type I events by time t. What is $E[N(t)|N_1(t+s) = j]$ for some integer $j \geq 0$?

2. Starting from time 0, the 99 buses arrive according to a rate λ Poisson process. Customers arrive according to an independent rate μ Poisson process. When a bus arrives, all waiting customers instantly enter the bus.

   (a) Find the PMF for the number of customers entering a bus.

   (b) Find the PMF for the number of customers entering the mth bus given that the interarrival time between Bus m and Bus $m - 1$ is t.

   (c) Suppose that the buses have been going for ever. You arrive at the bus stop at 8am. What is the expected length of time between the bus that comes before you arrived and the bus that you take?