Compressed Sensing (Math 555): Detailed outline

We learn the tools and concepts from Probability in High Dimensions which give foundational theory in Compressed Sensing and other Data Science problems.

This is a tentative outline.

1. Behaviour of sums of random variables, non-asymptotic deviation inequalities

    a. Sub-Gaussian random variables

    b. Sub-exponential random variables

    c. Bernstein inequality

2. Concentration of measure

    a. Concentration on the sphere and in Gauss space

    b. Implication: Lipschitz functions concentrate around their median

    c. Application: Johnson-Lindenstrauss lemma

3. Non-asymptotic random matrix theory and extrema of stochastic processes

    a. Review of asymptotic random matrix theory (this is given for an intuition, key results will be stated without proofs)

    b. Connection of random matrices to stochastic processes

    c. Covering arguments

    c. Slepian inequality, Gordon inequality

    d. Matrix Bernstein inequality

    e. Application: Covariance estimation

    f. Involved covering arguments: Dudley inequality, Generic chaining

    g. Application in compressed sensing: Restricted isometry property for sub-sampled Fourier transform

    h. Majorizing measures theorem

    i. Conditioning of a random matrix restricted to a fixed set, with applications in generalized compressed sensing, including compressed sensing with neural nets

    j. Fano's inequality (without information theory), showing optimality of compressed sensing results

4. More applications

    a. Matrix completion

b. Machine learning (generalization error bounds based on VC dimension)