**Probability in high dimensions (Math 608): Detailed outline**

This is a tentative outline. Roughly the following topics will be covered in roughly the following order. This may be revised based on class interest. A running application of interest is compressed sensing.

1. Behaviour of sums of random variables, non-asymptotic deviation inequalities

1. Sub-Gaussian random variables
2. Sub-exponential random variables
3. Bernstein inequality

2. Concentration of measure

a. Concentration on the sphere and in Gauss space.

b. Implication: Lipschitz functions concentrate around their median.

c. Application: Johnson-Lindenstrauss lemma

3. Non-asymptotic random matrix theory: extreme singular values

a. Review of asymptotic random matrix theory (this is given for an intuition, key results will be stated without proofs)

b. Connection to stochastic processes, simple covering arguments

c. Slepian inequality, Gordon inequality

d. Matrix Bernstein inequality

e. Application: Covariance estimation

f. Involved covering arguments: Dudley inequality, Generic chaining.

g. Application in compressed sensing: Restricted isometry property for sub-sampled Fourier transform

h. Decoupling. Matrices with random, independent columns

4. Geometric functional analysis

a. Gordon's escape through the mesh theorem, with application in compressed sensing

b. Random vectors drawn from convex bodies

d. Dvoretzky Milman theorem

d. Sudakov inequality

e. Low M\* estimate

f. Sections of l\_1 ball

g. Application: Optimality of theory of compressed sensing.

5. More applications

a. Open problem: Generalize escape through the mesh to give general theory of compressed sensing. We will describe the problem and some ideas we may use to solve it.

b. Lasso analysis, including lasso with non-linear measurements

c. Matrix completion

d. 1-bit compressed sensing

e. Constrained maximum-likelihood estimation