Compressed Sensing (Math 555): Detailed outline

We develop tools and concepts from high-dimensional probability that underlie the mathematical theory of compressed sensing and related problems in data science. Not all results will be proven; proofs are selected for insight and useful techniques.

This is a tentative outline. We may not cover all material below.

0. Compressed sensing and the restricted isometry property

1. Behaviour of sums of random variables, non-asymptotic deviation inequalities

   a. Sub-Gaussian random variables

   b. Sub-exponential random variables

   c. Bernstein inequality

2. Concentration of measure

   a. Concentration on the sphere and in Gaussian space

   b. Implication: Lipschitz functions concentrate around their median

   c. Application: Johnson-Lindenstrauss lemma

3. Non-asymptotic random matrix theory and extrema of stochastic processes

   a. Connection of random matrices to stochastic processes

   b. Covering arguments

   c. Slepian inequality, Gordon inequality

   d. Matrix Bernstein inequality

   e. Application: Covariance estimation

   f. Advanced covering arguments: Dudley inequality, Generic chaining

   g. Application in compressed sensing: Restricted isometry property for sub-sampled Fourier matrices

   h. Majorizing measures theorem

   i. Conditioning of a random matrix restricted to a fixed set, with applications in generalized compressed sensing, including compressed sensing with neural nets

   j. Fano's inequality (without information theory), showing optimality of compressed sensing results

4. More applications

   a. Matrix completion

b. Generalization error bounds in machine learning