Math 223 Assignment 9 Due Wednesday, April 5

We play with the singular value decomposition (svd). You may find the svd in fourth or fifth editions of your textbook (but unfortunately, not third), which is available at UBC library.

1. Let $Q, W \in \mathbf{R}^{5\times 5}$ be orthogonal matrices. Let q_1, q_2, q_3, q_4, q_5 be the columns of Q and w_1, w_2, w_3, w_4, w_5 be the columns of of W. To be precise q_i is the ith column of Q and w_i is the ith column of W. Give a basis for the null space of the following matrices, which are *almost* in SVD form:

   (a) (5 pts)
   $$A = Q\Sigma W^T, \qquad \Sigma = \begin{bmatrix} 10 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

   (b) (5 pts)
   $$B = QSW^T, \qquad S = \begin{bmatrix} 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & -5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2. Let A be a matrix with SVD $A = U\Sigma V^T$ (this is the standard SVD, not the reduced SVD). Suppose that
   $$\Sigma = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \end{bmatrix}.$$
   A *right-inverse* of A is a matrix B which satisfies $AB = I$. Does A have two or more right inverses? If so, find two right inverses. If not, why not?

   Hint: You may write your answer to the above question in terms of U and V. For example, if the question had been, *what is A^TA?* Then the answer would have been
   $$A^TA = V\Sigma^T\Sigma V^T = V \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 9 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} V^T.$$

3. (10 pts) (This question is inspired by the idea of *noise shaping* – the fact that in some applications one can force the noise to take a certain pattern.) Suppose that you have data from the (noiseless) linear model, but it becomes corrupted in the following way: One fixed, but unknown, constant is added to each entry of the data. To be precise, assume the following model:
   $$y = Ax + z, \qquad z = \begin{bmatrix} c \\ c \\ c \\ c \end{bmatrix}.$$

The matrix A has the singular value decomposition

$$A = U\Sigma V^T, \qquad U = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad \Sigma = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \qquad V \in \mathbf{R}^{3\times 3} \text{ is orthogonal.}$$

You observe y and A, but not c or x.

Is there a linear way to determine x? To be precise, is there a matrix W satisfying $Wy = x$ no matter what c is? If so, determine the matrix W (the answer could be written in terms of V).