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Math 223 - Midterm 1 - Friday October 3, 2008 - six pages

1.[20 marks] Using Gaussian Elimination, give all the solutions to the following system of equations in vector parametric form

$$\begin{array}{cccccccc} 2x_1 & +4x_2 & +2x_3 & +6x_4 & -2x_5 & = & 10 \\ 4x_1 & +8x_2 & +7x_3 & +15x_4 & -10x_5 & = & 26 \\ 2x_1 & +4x_2 & -4x_3 & & +10x_5 & = & -2 \end{array}$$

2.Given

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 0 \\ -2 & 2 & 4 \end{bmatrix}$$

a)[10 marks] Determine the eigenvalues for A. Hint: 2 is an eigenvalue.

b)[15 marks] Determine an eigenvector for each eigenvalue. If there is a repeated eigenvalue (a repeated root), determine two eigenvectors which are not multiples of one another.

3.[10 marks] Assume A satisfies $AM = MD$ where

$$A = \begin{bmatrix} 24 & 10 \\ -30 & -11 \end{bmatrix}, M = \begin{bmatrix} 1 & -2 \\ -2 & 3 \end{bmatrix}, D = \begin{bmatrix} 4 & 0 \\ 0 & 9 \end{bmatrix}.$$

If we set

$$C = \begin{bmatrix} 4^{3/2} & 0 \\ 0 & 9^{3/2} \end{bmatrix},$$

then $C^2 = D^3$. Use this fact to find an explicit matrix B so that $B^2 = A^3$ (I want the entries explicitly). Explain (displaying B is insufficient since checking this directly seems difficult).

4. [12 marks] For what values of the variable x does the matrix product AB have an inverse when A and B are given as follows?

$$A = \begin{bmatrix} x & 17 & 15 \\ 0 & 2 & 20 \\ 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & x & 1 \\ x & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

5.[5 marks] We have a function $f : \mathbf{R}^2 \longrightarrow \mathbf{R}^2$ with

$$f\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}, f\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}, f\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \end{pmatrix}.$$

Can f be a linear transformation? If yes, what is the associated matrix. If no, give a reason.

6.[8 marks] Assume A, B are two 3×3 matrices with the property that there exist an invertible matrix M and two diagonal matrices D_A, D_B so that $AM = MD_A$ and $BM = MD_B$. Show that $AB = BA$.

7.[10 marks] Consider a system of 4 equations. Consider the following row operation that someone has proposed: For the system of equations, replace the old equation 1 with the sum of the old equation 1 and -1 times the old equation 2 and simultaneously replacing the old equation 2 with the sum of the old equation 2 and -1 times the old equation 1. Does this preserve the set of solutions to the system of equations?

8.[10 marks] Let A be a 4×4 matrix with an eigenvalue of 2. Show that $A^2 - 3A + 2I$ is not invertible.