Ex) Find $f'(0)$ where $f(x) = \arctan(x)$
$= \tan^{-1}(x)$.

$$f'(x) = \frac{1}{1+x^2}$$

$$\Rightarrow \boxed{f'(0) = 1}$$

Ex) Find $f'(0)$ where $f(x) = \arctan(5x)$

$$f'(x) = \frac{1}{1+(5x)^2} \cdot 5$$

$$\Rightarrow \boxed{f'(0) = 5}$$

Ex) Find $\frac{dy}{dx}$ where $y = \log(\sin^{-1}(\log x))$

$$\boxed{\frac{dy}{dx} = \frac{1}{\sin^{-1}(\log(x))} \cdot \frac{1}{\sqrt{1-(\log x)^2}} \cdot \frac{1}{x}}$$

Ex) Find $f'(x)$ where $f(x) = \dfrac{(x^2+3\sin^2(x))\, e^{x^2}}{x^4+7}$

Take ln of both sides:

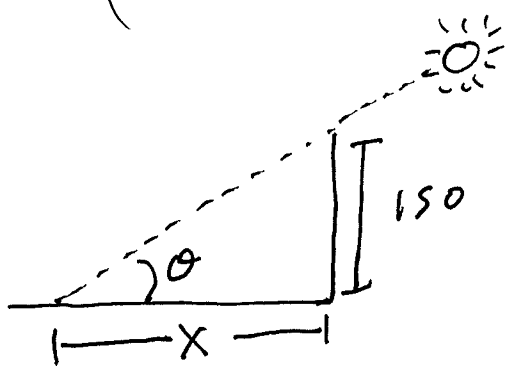
$$\ln f(x) = \ln(x^2+3\sin^2(x)) + \underbrace{\cancel{\ln(e^{x^2})}}_{x^2} - \ln(x^4+7)$$

$\frac{d}{dx}$ both sides:

$$\frac{f'(x)}{f(x)} = \frac{2x + 6\sin(x)\cos x}{x^2+3\sin^2(x)} + 2x - \frac{4x^3}{x^4+7}$$

$$\Rightarrow \boxed{f'(x) = \left(\frac{2x + 6\sin(x)\cos(x)}{x^2 + 3\sin^2(x)} + 2x - \frac{4x^3}{x^4+7} \right) \frac{(x^2 + 3\sin^2(x))\, e^{x^2}}{x^4+7}}$$

Ex) (Example 3 from Section 3.10)



a) Express θ as a function of x.

$$\tan(\theta) = \frac{150}{x}$$

$$\Rightarrow \boxed{\theta = \tan^{-1}\left(\frac{150}{x}\right)}$$

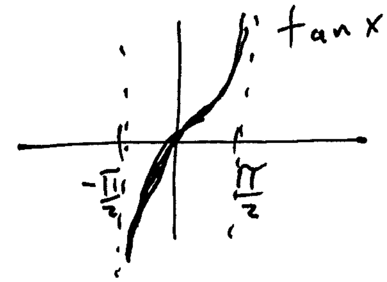
b) Compute $\frac{d\theta}{dx}$ when $x = 200$ ft.

$$\boxed{\frac{d\theta}{dx} = \frac{d}{dx} \tan^{-1}\left(\frac{150}{x}\right) = \frac{1}{1 + \left(\frac{150}{x}\right)^2} \cdot \frac{-150}{x^2}}$$

$$\boxed{\left.\frac{d\theta}{dx}\right|_{x=200} = \frac{1}{1 + \left(\frac{150}{200}\right)^2} \cdot \frac{-150}{200^2}}$$

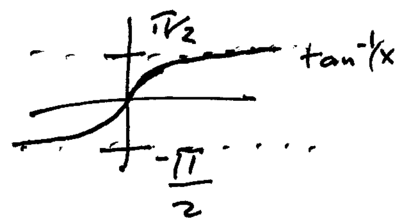
$\frac{d\theta}{dx}$ measures how the angle of the sun changes as we vary x.

Ex) Graph $f(x) = \tan^{-1}(x^2)$.



<u>Step 1:</u> Domain: $(-\infty, \infty)$

<u>Step 2:</u> Asymptotes: $\lim_{x \to \infty} \tan^{-1}(x^2) = \frac{\pi}{2}$



$$\lim_{x \to -\infty} \tan^{-1}(x^2) = \frac{\pi}{2}$$

<u>Step 3:</u> Crit pts.

$$f'(x) = \frac{1}{1+(x^2)^2} \cdot 2x = \frac{1}{1+x^4} \cdot 2x$$

Always defined so set $f'(x) = 0$ to find crit pts.

$\Rightarrow 0 = \frac{1}{1+x^4} \cdot 2x \Rightarrow \boxed{x = 0}$ is the crit pt

$$\text{sign}(f'(x)) = \text{sign}\left(\frac{1}{1+x^4} \cdot 2x\right) = \text{sign}(x).$$

<u>Step 4:</u> Inflection pts and sign(f'').

$$f''(x) = \frac{d}{dx} \frac{2x}{1+x^4} = \frac{(1+x^4) \cdot 2 - 2x \cdot 4x^3}{(1+x^4)^2}$$

$$= \frac{2 + 2x^4 - 8x^4}{(1+x^4)^2}$$

$$= \frac{2(1-3x^4)}{(1+x^4)^2} = \frac{6\left(\frac{1}{3} - x^4\right)}{(1+x^4)^2}$$

Inflection pts may exist where

$$f''(x) = 0 \Rightarrow \frac{1}{3} - x^4 = 0 \Rightarrow x = \pm\left(\frac{1}{3}\right)^{\frac{1}{4}}$$

$$\operatorname{sign}(f''(x)) = \operatorname{sign}\left(\frac{6\cdot\left(\frac{1}{3}-x^4\right)}{(1+x^4)^2}\right) = \operatorname{sign}\left(\frac{1}{3}-x^4\right) = \begin{cases} - & \text{if } x^4 > \frac{1}{3} \\ + & \text{if } x^4 < \frac{1}{3} \end{cases}$$

$$= \begin{cases} - & \text{if } |x| > \left(\frac{1}{3}\right)^{\frac{1}{4}} \\ + & \text{if } |x| < \left(\frac{1}{3}\right)^{\frac{1}{4}} \end{cases}$$

Step 5 Tabulate

x	$-\infty$	$(-\infty, (\frac{1}{3})^{1/4})$	$-(1/3)^{\frac{1}{4}}$	$(-(\frac{1}{3})^{\frac{1}{4}}, 0)$	0	$(0, (\frac{1}{3})^{\frac{1}{4}})$	$(1/3)^{1/4}$	$((\frac{1}{3})^{\frac{1}{4}}, \infty)$	
f'		$-$		$-$		$+$		$+$	
f''		$-$		$+$		$+$		$-$	
f	$f \to \frac{\pi}{2}$		infl.		$f' = 0$ $f(0) = 0$		infl.		$f \to \pi/2$

Step 6: Graph: