Find equation for tangent line to curve

$$x^3 + xy^2 + y^3 = 13 \quad \text{at } (1,2).$$

$$\frac{dy}{dx}\left(x^3 + xy^2 + y^3\right) = \frac{d}{dx}13 = 0$$

$$= 3x^2 + 1\cdot y^2 + x\cdot 2y\cdot\frac{dy}{dx} + 3y^2\frac{dy}{dx}$$

Plug in $(1,2)$:

$$3\cdot 1^2 + 1\cdot 2^2 + 1\cdot 2\cdot 2\,\frac{dy}{dx} + 3\cdot 2^2\,\frac{dy}{dx} = 0$$

$$\Rightarrow 7 + 16\frac{dy}{dx} = 0$$

$$\Rightarrow \boxed{\frac{dy}{dx} = -\frac{7}{16}}$$

Tan line:

$y = -\frac{7}{16}x + b$ Plug in $(1,2)$: $2 = -\frac{7}{16}\cdot 1 + b$

$$\Rightarrow \frac{32}{16} = -\frac{7}{16} + b$$

$$\Rightarrow b = \frac{39}{16} \Rightarrow \boxed{y = -\frac{7}{16}x + \frac{39}{16}}$$

Find slope of tangent line to curve

$$\sqrt{x+y^2} = 3xy - 7 \quad \text{at pt } (3,1).$$

$$\frac{d}{dx}\sqrt{x+y^2} = \frac{d}{dx}(3xy-7)$$

$$\Rightarrow \frac{1}{2}(x+y^2)^{-\frac{1}{2}}\frac{d}{dx}(x+y^2) = 3y + 3x\frac{dy}{dx} - 0$$

$$\Rightarrow \frac{1}{2}(x+y^2)^{-\frac{1}{2}}\left(1 + 2y\frac{dy}{dx}\right) = 3y + 3x\frac{dy}{dx}$$

Plug in $(3,1)$:

$$\frac{1}{2}(3+1^2)^{-1/2}\left(1 + 2\cdot 1\frac{dy}{dx}\right) = 3\cdot 1 + 3\cdot 3\cdot\frac{dy}{dx}$$

$$\Rightarrow 4\cdot\frac{1}{2}\cdot\frac{1}{2}\left(1+2\frac{dy}{dx}\right) = \left(3 + 9\frac{dy}{dx}\right)\cdot 4$$

$$\Rightarrow \quad 1 + 2\frac{dy}{dx} = 12 + 36\frac{dy}{dx}$$

$$\Rightarrow -11 = 34\frac{dy}{dx} \quad \Rightarrow \boxed{\frac{dy}{dx} = \frac{-11}{34}}$$