1. $$\lim_{x\to 2} \frac{x^2+x-6}{3x^2-5x-2} = \frac{p(x)}{q(x)}$$

$q(x) = 3x^2-5x-2$

$q(2) = 3\cdot 4 - 5\cdot 2 - 2 = 0$

$\Rightarrow$ Can't use Thm 2.4

$$= \lim_{x\to 2} \frac{\cancel{(x-2)}(x+3)}{\cancel{(x-2)}(3x+1)}$$

$$= \frac{2+3}{3\cdot 2+1} = \boxed{\frac{5}{7}}$$

2. $$\lim_{x\to 2} \frac{\frac{x}{x^2+1} - \frac{2}{5}}{x-2} \cdot \underbrace{\frac{5(x^2+1)}{5(x^2+1)}}_{1}$$

$$= \lim_{x\to 2} \frac{5x-2(x^2+1)}{(x-2)\cdot 5(x^2+1)}$$

$$= \lim_{x\to 2} \frac{-2x^2+5x-2}{(x-2)5(x^2+1)}$$

$$= \lim_{x\to 2} \frac{\cancel{(x-2)}(-2x+1)}{\cancel{(x-2)}5(x^2+1)}$$

$$= \frac{-2\cdot 2+1}{5\cdot(2^2+1)} = \boxed{\frac{-3}{25}}$$

(3) $$\lim_{x \to 16} \frac{\sqrt{x} - 4}{x - 16} \cdot \frac{\sqrt{x} + 4}{\sqrt{x} + 4}$$

$$= \lim_{x \to 16} \frac{\cancel{x-16} \cdot 1}{\cancel{x-16}(\sqrt{x} + 4)}$$

$$= \frac{1}{\sqrt{16} + 4} = \boxed{\frac{1}{8}}$$

$$(a+b)(a-b) = a^2 - ab + ab - b$$
$$= a^2 - b^2$$

(4) $$\lim_{x \to 0} \frac{(\sqrt{x+1} - 1)}{x} \frac{(\sqrt{x+1} + 1)}{(\sqrt{x+1} + 1)}$$

$$= \lim_{x \to 0} \frac{\cancel{x}\cancel{+1}\cancel{-1}\ 1}{\cancel{x}(\sqrt{x+1} + 1)}$$

$$= \frac{1}{\sqrt{0+1} + 1} = \boxed{\frac{1}{2}}$$

5 $$\lim_{x\to 4} \frac{x^2-3x-4}{\sqrt{x}-2} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2}$$

$$= \lim_{x\to 4} \frac{(x^2-3x-4)(\sqrt{x}+2)}{x-4}$$

$$= \lim_{x\to 4} \frac{\cancel{(x-4)}(x+1)(\sqrt{x}+2)}{\cancel{(x-4)}}$$

$$= \lim_{x\to 4}(x+1) \cdot \lim_{x\to 4}(\sqrt{x}+2)$$

$$= (4+1) \cdot \left(\lim_{x\to 4}\sqrt{x} + \lim_{x\to 4} 2\right)$$

$$= \boxed{5 \cdot (\sqrt{4}+2) = 5 \cdot 4 = \boxed{20}}$$